Entrepreneurship, Financiership, and Selection*

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Abstract

We develop an equilibrium model of the market for entrepreneurial finance where all agents are endowed with some personal wealth and a project whose quality is their private information. All agents are capital constrained and need to choose whether to invest as entrepreneurs or financiers, or in the storage technology. We find that a binding economy-level wealth constraint making credit scarce tends to improve efficiency by raising the cost of capital and creating advantageous selection where the agents with productive projects become entrepreneurs and those with unproductive ones become their financiers. If funding is easier to come by, entrepreneurship becomes attractive also for unproductive agents. Injecting capital into the financial market may therefore have harmful efficiency effects due to adverse selection. In our model individual wealth and entrepreneurship are positively (negatively) correlated if financial market participation is complete (incomplete). Even if insufficient individual wealth holds back business creation, there can be too much entrepreneurship.

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1. Introduction

It has been widely recognized that innovative start-ups are engines of job creation and economic growth, but that the market for entrepreneurial finance is fraught with market failures. These observations have lead to extensive public intervention with the aim of promoting entrepreneurship and the finance thereof. However, as evidence surveyed by Lerner (2009) suggests, policies spurring entrepreneurship often fail for reasons that are not completely understood. To shed light on the challenges faced by policymakers and the role of different market imperfections, we construct a simple equilibrium model of entrepreneurial finance that incorporates major perceived reasons for market failures: Asymmetric information, individual and aggregate capital constraints.

A key message emerging from our paper is that many public interventions will simultaneously affect both entrepreneurs' and financiers' outside options and may hence have unintended consequences. For example, an injection of capital into a market where some productive projects cannot be executed due to lack of credit will simultaneously make entrepreneurship more and its finance less attractive and may thus lead to adverse selection.

We also identify an aggregate wealth constraint as a crucial determinant of market efficiency: When the total wealth of the economy is sufficient to implement all projects with a positive net present value, interest rates tend to be low enough to attract agents with low quality projects to entrepreneurship. In contrast, a binding economy-level wealth constraint can induce advantageous selection. When credit is restricted, higher interest rates simultaneously discourage entrepreneurship and encourage its finance so that agents with low quality projects prefer financing others' ventures to investing in their own projects.

Another implication of our model concerns the relationship between agents' wealth and entrepreneurship. A large empirical literature has documented a positive relationship between individual wealth and entrepreneurship (e.g., Evans and Jovanovic 1989, Black et al., and Gentry and Hubbard 2004), whereas the basic adverse selection theory predicts the reverse (e.g., de Meza and Webb 1987). Our model, too, shows that individual wealth and entrepreneurship can be negatively correlated, but only if there is so much wealth in the economy that some agents opt out of the financial markets. If financial market participation is complete, there is a positive relationship between individual wealth and entrepreneurship.[1] However, in our model there can be too much entrepreneurship even if individual wealth and entrepreneurship are positively correlated.

We build on the well-established literature on entrepreneurial finance with asymmetric information emerging from Stiglitz and Weiss (1981) and de Meza and Webb (1987). De Meza and Webb (e.g., 1987, 1990, and 1999) argue that the existence of credit constraints or asymmetric information as such cannot be used as rationales for subsidizing entrepreneurship or the finance thereof.[2] Here the same conclusion emerges as part of equilibrium when the agents' individual wealth levels are moderate and the economy-level wealth constraint does not bind. When it binds, however, increases in individual wealth cause entry of productive entrepreneurs and a case for subsidizing business creation may arise. Like us, de Meza and Webb (1999) show that excessive entrepreneurial activity can co-exist with a positive relationship between individual wealth and entrepreneurship, but here coexistence arises without moral hazard considerations.

[1] In other words, our model predicts that it can be difficult to establish an unambiguous correlation between individual wealth and entrepreneurship. This is in line with Hurst and Lusardi (2004) who show that wealth and entrepreneurship are hardly positively related across all wealth classes.

[2] For an elegant generalization of this argument, see Boadway and Keen (2006).

Our paper also ties with the literature on occupational choice in the presence of financial market frictions. In particular, the contemporaneous works by Inci (2006) and Ghatak et al. (2007) consider general equilibrium models where privately informed agents facing credit constraints choose between entrepreneurship and paid employment.[3] We focus on the financial market participation decision and the choice between becoming an entrepreneur and a financier. Our relatively sparse model yields a rich set of outcomes and policy implications. Even without labor market considerations, our model generates a similar outside option mechanism as in Inci (2006) and Ghatak et al. (2007). This mechanism drives selection into entrepreneurship and, in our case, into 'financiership'. Moreover, we identify an economy-wide wealth constraint which determines whether selection creates adverse or advantageous effects.

To the best of our knowledge, Boyd and Prescott (1986) and Shleifer and Wolfenzon (2002) are the only studies besides ours where there is a genuine choice between investing as an entrepreneur and a financier. Whereas the focus in Shleifer and Wolfenzon (2002) has little to do with our analysis, Boyd and Prescott (1986) is close to our study. Our model is simpler than theirs in that we do not allow for financial intermediaries that produce information. Unlike us, Boyd and Prescott (1986) focus on an economy where the aggregate wealth constraint is not binding. We show that in many cases when aggregate wealth constraint binds or agents are rich enough, there is no need for information provision by financial institutions since the markets are efficient. We also highlight the comparative statics over agents' wealth.

[3] Yet another related contemporaneous study is Antunes et al. (2008) where agents differing in their managerial talent (which is common knowledge) not only choose between paid employment and entrepreneurship but also whether they invest their initial wealth in their own firm or in the others' firms via financial intermediaries. They focus on explaining cross-country variations in economic performance by exogenous intermediation costs and investor protection.

Besides the aforementioned articles, our study is also inspired by Holmström and Tirole (1997, 1998), Caballero and Krishnamurthy (2001) and Aghion et al. (2004) who emphasize that both micro- and economy-level financial constraints influence the performance of financial markets. From this perspective our study has also a link to the literature studying the effects of financial liberalization in the presence of adverse selection (e.g., Giannetti 2007 and Sengupta 2007).

In the next Section, we present the model. In Section 3, we show how to construct equilibria, and analyze their existence and efficiency while relegating most of the algebra in the Appendices. To clarify the contribution of our paper and to provide some intuition for our results, we compare our model to the standard partial equilibrium model with unlimited supply of finance. We analyze the relationship between wealth and entrepreneurship in Section 4, and Section 5 is devoted to the policy implications. Section 6 concludes.

2. The Model

The economy is populated by a [0,1] continuum of risk-neutral agents who have access to a project of size I and have personal wealth (cash) $0<A<I$. The projects have a two-point return distribution: A proportion h ($0<h<1$) of agents are high (H) types who are endowed with a positive NPV project, the rest are low (L) types with a negative NPV project. As, e.g., in Holmström and Tirole (1997) we assume that $p_H R_H > I > p_L R_L$ and $R_L > R_H$, where p_t is the success probability and R_t the return (conditional on success) of an entrepreneur of type t, $t \in \{H, L\}$. Failed projects yield zero. In words, project return distributions are characterized by second-order stochastic dominance (but not mean-preserving spread).[4] Following the convention in the literature (e.g., de Meza and Webb 1987 and Boadway and Keen 2006), we

[4] In Section 3.3 we briefly consider the first-order stochastic dominance and mean-preserving spread of project return distributions ($R_H=R_L$ and $p_H R_H=p_L R_L$, respectively).

assume that agents are protected by limited liability, agents' personal wealth is common knowledge but that project type is their private information.

In our model agents can choose whether they i) become entrepreneurs, investing their wealth in their own project,[5] ii) become financiers, investing their wealth to finance the projects of others, or iii) invest their wealth in the storage technology. That is, each agent chooses action $a \in \{e, f, s\}$ from the agents' common action space with *e*, *f*, and *s* denoting entrepreneurship, 'financiership', and storage technology, respectively.

As entrepreneurs need to raise external funds from other potential entrepreneurs, the agents' aggregate initial wealth constrains investment possibilities. We consider an economy *wealth constrained* if the total wealth of all agents $A_{agg} \equiv A\int_0^1 di$ is insufficient to finance all H-type projects $hI_{agg} \equiv hI\int_0^1 di$, i.e., if $A_{agg}/I_{agg}=A/I<h$. Correspondingly, an economy is *non-wealth constrained* if the total wealth exceeds the financing needs of all H-type projects (i.e., if $A_{agg}/I_{agg}=A/I\geq h$).

We allow no financial institutions processing information, and so the financial market in our model could be interpreted as a frictionless (credit) market, a mutual fund, or a microfinance institution.[6] The market collapses to *autarky* when all agents resort to the storage technology and there are neither entrepreneurs nor financiers. In *an efficient equilibrium* all or as many H-type projects as possible are financed whereas no L-type projects receive finance. Correspondingly, in *an inefficient equilibrium* at least some L-type projects are carried out.

[5] Agents becoming entrepreneurs must invest their wealth entirely in their own project in equilibrium. As, e.g., in de Meza and Webb (1987), it is cheaper for H-type entrepreneurs to use their own rather than others' funds. Consequently, L-type entrepreneurs have no other option but to follow and invest all their wealth in their own projects. Note that the agents have no illiquid outside wealth that could be pledged as a collateral to facilitate the emergence of a separating equilibrium as, e.g., in Bester (1987).

[6] A more guarded interpretation is a group within a microfinance institution but even the formation of such groups can be thought of as being frictionless (see Eeckhout and Munshi 2007).

We focus on risky debt contracts that give a financier a fixed repayment in case of success and zero otherwise.[7] Following some other common practices in the literature, we assume that the storage technology is perfect with a zero rate of return,[8] that markets must clear in equilibrium, and that agents cannot publicly destroy their individual wealth before investing.

In sum, the timing of events is as follows: First, each agent decides whether to invest her individual wealth (A) in her own project, in the projects of others, or in storage. If she initiates her own project, the rest of required funds (I-A) needs to be raised from the agents who became financiers. Debt contract terms stipulate the conditional payment from the entrepreneur to investors in case of success. Second, entrepreneurs execute their projects. Project returns are realized and successful entrepreneurs compensate financiers according to the debt contract.

3. Equilibria

We look for Bayesian equilibria where agents' pure strategies are functions from their common type space $\{H,L\}$ into their common action space $\{e,f,s\}$, and agents correctly anticipate such type-contingent strategies of the other agents. It is possible that at least some agents randomize over their pure strategies in equilibrium. Since there is a continuum of agents, we model mixed strategies by a distributional approach where a proportion μ_t of t-type agents use the pure strategy of becoming an

[7] The focus on debt financing in the literature is often motivated by its prevalence in practice. We share this motivation: e.g., according to the European Commission (2009) and Robb and Robinson (2009) debt constitutes the main form of external finance for start-ups. But note that risky debt contracts can also be optimal in plausible circumstances, e.g., when only payments from entrepreneurs to financiers are verifiable and entrepreneurs cannot hide income in case they default (see, e.g., de Meza and Webb 1999), or when project success is verifiable but project returns are not (or when returns are verifiable at most up to R_H as, e.g., in Bolton and Scharfstein 1990) and a debt contract cannot specify a positive reward to refrain from investing. Studying the consequences of richer verifiability assumptions for optimal security design in our set up where agents can choose their financial occupation and economy-level wealth constraint can bind is left for the future research. However, at least to the extent the equilibria we investigate are efficient, there is no room for more efficient contracting even under richer verifiability assumptions.

[8] In a longer working paper version (Takalo and Toivanen 2006) we allow for imperfect storage technology.

entrepreneur, proportion χ_t uses the pure strategy of investing in storage, and proportion 1-μ_t-χ_t uses the pure strategy of becoming a financier for some μ_t, $\chi_t \in [0,1]$, $t \in \{H,L\}$. In equilibrium, proportions μ_t and χ_t satisfy the indifference requirements between the payoffs from the pure strategies. The expected profits of an agent of type t from action a are denoted by $\pi(a,t)$.

The equilibria consist of four conditions. The first arises from individual rationality (IR) constraints: All agents compare the expected profits from participating in the financial market, either as an entrepreneur or as a financier, to investing in storage. Since the efficiency of storage technology is independent of the agent's type (i.e., $\pi(s,H) = \pi(s,L) = A$), the IR constraints can be written as

(1) $\pi(a,t) \geq A,\ a \in \{e,f\},\ t \in \{H,L\}.$

Second, the incentive compatibility (IC) constraints guide the agent's choice between investing as an entrepreneur and as a financier:

(2) $\pi(a,t) \geq \pi(a',t),\ a,a' \in \{e,f\},\ a \neq a',\ t \in \{H,L\}.$

The third condition equalizes the supply of funds from financiers with the demand of funds by entrepreneurs:

(3) $(I_{agg} - A_{agg})[\mu_H h + \mu_L(1-h)] = A_{agg}[(1-\mu_H - \chi_H)h + (1-\mu_L - \chi_L)(1-h)].$

The left hand side of (3) captures the demand: Each entrepreneur demands I-A of funds, and the equilibrium mass of entrepreneurs is $[\mu_H h + \mu_L(1-h)]\int_0^1 di$. Similarly, the supply of funds from financiers is given by the right hand side of (3).

Finally, the (expected) payments by successful entrepreneurs must equal the (expected) payments received by financiers, i.e., it must hold that

(4) $R_B[\mu_H h p_H + \mu_L(1-h)p_L] = R_F[(1-\mu_H - \chi_H)h + (1-\mu_L - \chi_L)(1-h)].$

In (4) R_B is the fixed payment an entrepreneur has promised to pay back in case of success, and R_F is the expected payment received by a financier which, owing to the law of large numbers, equals the realized payment. The term in the square brackets on the right hand side of (4) is the equilibrium proportion of financiers, whereas the term on the left gives the equilibrium proportion of successful entrepreneurs.

In equilibrium, other agents' types and actions affect an agent's payoffs only through the cost of borrowing as an entrepreneur (R_B) and the repayment received as a financier (R_F). Hence, expected profits from entrepreneurship can be written as

(5) $$\pi(e,t) = p_t(R_t - R_B),\ t \in \{H,L\}$$

and from investing as a financier as

(6) $$\pi(f,t) = R_F,\ t \in \{H,L\}.$$

Table 1: Potential Equilibria

	$\mu_L = 0$	$0 < \mu_L < 1$	$\mu_L = 1$
$\mu_H = 0$	AUTARKY	Not possible	Not possible
$0 < \mu_H < 1$	$H^{ef} L^{f}$	$H^{ef} L^{ef}$	$H^{ef} L^{e}$
$\mu_H = 1$	$H^{e} L^{fs}$	$H^{e} L^{ef}$, $H^{e} L^{efs}$	Not possible

Notes:
μ_t denotes the proportion of *t* type agents becoming entrepreneurs in equilibrium.
Superscript *e* stands for entrepreneurs, *f* for financiers, *s* for users of storage.

In Table 1 we show a 3x3 matrix of potential equilibria.[9] It is immediately clear that three out of the nine cannot exist. If no H-type agent becomes an entrepreneur, the potential financiers' IR constraints are violated. Similarly, because $A_{agg} < I_{agg}$, it is impossible that all agents become entrepreneurs. The remaining six potential equilibria consist, besides autarky, of five cases where financial markets

[9] These nine categories can be split further according to whether agents participate or not.

emerge as an equilibrium outcome. We name these five according to what actions agents choose. For example, in equilibrium $H^e L^{fs}$ all H-type agents become entrepreneurs, and L-types split between becoming financiers and using storage.

Both efficient equilibria are in the first column of Table 1. The other three equilibria with financial markets are inefficient since at least some L-types become entrepreneurs. It is straightforward but tedious to solve the range of parameters where (1)-(4) hold for all five equilibria with financial markets. Hence we construct only the equilibrium $H^e L^{ef}$ in detail in the main text. The remaining equilibria are described graphically, with detailed calculations being presented in the Appendices.

3.1. Example: $H^e L^{ef}$

In $H^e L^{ef}$, $\mu_H = 1$, $\mu_L \in (0,1)$, and $\chi_H = \chi_L = 0$, i.e., all H-type agents are entrepreneurs, L-types become either entrepreneurs or financiers, and nobody invests in the storage technology. This outcome corresponds to the decentralized market equilibrium in Boyd and Prescott (1986). It is also similar to a pooling equilibrium familiar from the standard partial equilibrium model in the sense that there is no aggregate shortage of funds and L-types are inefficiently attracted to entrepreneurship.

Since financial market participation is complete in this equilibrium, we require that agents' IR constraints are satisfied, i.e., from (1) and (6) we get that

(7) $$R_F \geq A.$$

L-type agents split between the two occupations, so their IC constraint must hold with equality, which by (2), (5) and (6) means that

(8) $$p_L (R_L - R_B) = R_F \ .$$

The left hand side of (8) gives the expected return of an L-type agent from becoming an entrepreneur and the right hand side gives the expected return from becoming a financier.

Because all H-type agents prefer entrepreneurship to being financiers, their expected return from entrepreneurship must be at least as large as that of becoming a financier, i.e., by (2), (5) and (6) the following must hold:

(9) $$p_H(R_H - R_B) \geq R_F .$$

Inserting $\mu_H = 1$ and $\chi_H = \chi_L = 0$ into (3) and (4) shows that the aggregate demand and supply for entrepreneurial finance is balanced when

(10) $$(I_{agg} - A_{agg})\left[h + \mu_L(1-h)\right] = A_{agg}(1-\mu_L)(1-h),$$

and that the (expected) repayment from successful entrepreneurs equal the payments received by their financiers when

(11) $$R_B\left[hp_H + \mu_L(1-h)p_L\right] = R_F(1-\mu_L)(1-h) .$$

Conditions (8), (10) and (11) determine the endogenous variables μ_L, R_B, and R_F. Solving first for the proportion of L-type entrepreneurs μ_L from (10) gives $\mu_L^* = (A_{agg} - hI_{agg})/(1-h)I_{agg}$ which can be rewritten, after recalling that $A_{agg} = A\int_0^1 di$ and $I_{agg} = I\int_0^1 di$, as

(12) $$\mu_L^* = \frac{A - hI}{(1-h)I} .$$

Using (8), (11) and (12) we can solve for the equilibrium payments R_B^* and R_F^*:

(13) $$R_B^* = \frac{(I - A)}{I\left[hp_H + (1-h)p_L)\right]} p_L R_L \text{ and}$$

(14) $$R_F^* = \left[1 - \frac{p_L(I - A)}{I\left[hp_H + (1-h)p_L)\right]}\right] p_L R_L .$$

To check out the range of parameter values where H-type agents' IC constraint is satisfied, we substitute (13) and (14) for (9). This yields

(15) $$A \geq I - \frac{(p_H R_H - p_L R_L) I\left[hp_H + (1-h)p_L)\right]}{(p_H - p_L) p_L R_L} .$$

When (15) holds, the H-types' IR constraint is redundant, and we only need to ensure that L-types' IR holds. Inserting (14) into (7) gives

$$(16) \qquad A \leq \frac{p_L R_L I h(p_H - p_L)}{Ih(p_H - p_L) + p_L\left(I - p_L R_L\right)}.$$

When (16) holds, no L-type agent stores her individual wealth.

The set of parameter values for which $H^e L^{ef}$ exists is shown in Figure 1 in the (*A*, *h*) space where the horizontal axis represents agents' individual wealth ($A<I$) and the vertical axis the proportion of H-type agents ($h\leq 1$). The $h=A/I$ diagonal divides the economy into wealth constrained (above), and non-wealth constrained (below). By definition, $H^e L^{ef}$ can only exist in a non-wealth constrained economy where the aggregate wealth is sufficient to finance all H-types' projects. This can also be seen from (12): $\mu_L^* \geq 0$ implies that $A/I\geq h$. The H-type's IC constraint (15) is a downward sloping line in the (*A*, *h*)-space so that a relatively poor economy with a relatively small proportion of good projects fails to satisfy this constraint. The L-type agent's IR constraint (16) is a monotonically increasing curve that starts at the origin and cuts the diagonal once. Below the curve, some L-types prefer not to participate.

FIGURE 1 HERE

Let us consider how a small decrease in aggregate wealth affects the equilibrium outcome. Clearly, it reduces the funds available to entrepreneurs. Because in this equilibrium H-types prefer entrepreneurship to becoming a financier and L-types are indifferent, a marginal change in wealth has no impact on H-types' choices, but some L-types must exit from entrepreneurship and become financiers (μ_L^* declines). That is, the ratio of entrepreneurs to financiers diminishes, which drives the compensation per financier (R_F^*) downwards. Since each entrepreneur needs to borrow more, R_B^* increases just enough to keep L-types indifferent.

If aggregate wealth decreases sufficiently, it will inevitably also affect H-types' actions. If their proportion is high enough, L-types will continue to exit from entrepreneurship until the market runs out of funds. When this happens, all L-types are financiers, i.e., we hit the $\mu_L{}^* \geq 0$-constraint in Figure 1, and some H-types need to begin to supply funding. If the proportion of H-types is low enough, the changes R_F^* and R_B^* break the H-types' IC constraint (15) before the $\mu_L{}^* \geq 0$-constraint.

The example illustrates how a decrease in aggregate wealth tightens the financial market, which is beneficial from the efficiency point of view. Like in de Meza and Webb (1987), marginal entrepreneurs are of low quality and are driven out by higher lending rates. But here the problem of overinvestment emerges as part of equilibrium: The economy's total wealth relative to the proportion of high quality projects is too large. Moreover, there is a positive relationship between an individual agent's wealth and entrepreneurship. This is at odds with the prediction of de Meza and Webb (1987), but has empirical appeal.

3.2. Existence and Efficiency of Equilibria

Following the procedure outlined in the previous Section, we analyze the existence of the remaining equilibria where financial markets endogenously emerge in the Appendices. Here we present graphically the results and describe their efficiency properties. At the end of the Section, we summarize our main results.

In Figure 2, we indicate the areas in which each equilibrium exists in the (A, h)-space. There are two key lines: the $h=A/I$-diagonal and the vertical $\hat{A} \equiv p_L p_H (R_L - R_H)/(p_H - p_L)$-line. These are shown at full length, with the sections confining equilibria in bold. The other bold lines bounding equilibria come from agents' various IC and IR constraints. For example, in the middle of a non-wealth constrained region we have a part of the L-type IR constraint (16) familiar from the case H^eL^{ef} of Section 3.1. Similarly, Figure 2 shows the part of H-types' IC constraint (15) that act as a boundary of equilibria.

As Figure 2 illustrates, the diagonal not only determines whether the economy is wealth constrained or not, but also borders equilibria in many cases. To the left of the vertical $\hat{A}$-line, limited liability makes the L-type agents' pledgeable income higher than H-type agents'.[10] To the right of the $\hat{A}$-line (where H-type agents' pledgeable income is higher than L-type agents'), the equilibria are unique.[11]

FIGURE 2 HERE

Let us first examine a non-wealth constrained economy (below the diagonal in Figure 2). In the right part we find an efficient equilibrium, H^eL^{fs}. All H-type projects are financed, and L-types are indifferent between funding the H-types and investing in the storage technology. If $A \geq p_L R_L$, L-types trivially prefer investing in storage to entrepreneurship. Because of costly financing ($R_B^* > 0$), L-types continue to find storage superior even if their individual wealth somewhat falls below the expected return of their project.

When entrepreneurs have a sufficiently low stake in their projects (once A falls below the vertical $\overline{A}$-line ($\overline{A} \equiv p_L(p_H R_L - I)/(p_H - p_L) < p_L R_L$), L-types are no longer discouraged from entrepreneurship. This results in an inefficient semi-separating equilibrium, where some L-type agents pool with H-types as entrepreneurs. Below the L-type IR-curve (16), we have the equilibrium H^eL^{efs} where some L-types opt for storage. Above the L-type IR-curve, we find H^eL^{ef} of Section 3.1. Nobody uses storage even though available assets exceed the H-type entrepreneurs' funding

[10] Pledgeable income, as defined by Holmström and Tirole (1997), is the maximum amount an entrepreneur can credibly promise to pay back to a financier.

[11] Unique under our assumptions which, e.g., postulate that initial wealth cannot be publicly destroyed and markets must clear.

needs, so demand and supply of funds is equated through some L-types becoming entrepreneurs. Autarky prevails to the left of the H-types' IC constraint (15).

In a wealth-constrained economy (above the diagonal in Figure 2), the outcomes are quite different. In the middle and right part, characterized by wealthier agents with a higher fraction of good projects, all L-types are financiers and H-types mix between entrepreneurs and finance. This $H^{ef}L^{f}$ equilibrium is efficient: the total endowment of the economy is directed into positive net present value projects.

Entrepreneurship becomes relevant to L-types only when we reach the vertical $\hat{A}$-line. Recall that here the relative size of agents' plegeable income changes so that to the left of the line, L-types have a higher pledgeable income. Between it and another vertical line, $\hat{A}I/p_H R_H$, we have one to three equilibria. One is the same $H^{ef}L^{f}$ as to the right of the $\hat{A}$-line. There are also two inefficient equilibria: $H^{ef}L^{ef}$ where both agent types can be found among entrepreneurs and financiers, and $H^{ef}L^{e}$ where all L-types are entrepreneurs and H-types split between entrepreneurship and being a financier. Interestingly, only $H^{ef}L^{e}$ survives to the left of the $\hat{A}(I/p_H R_H)$-line. The equilibrium can be supported in an area where the proportion of H-type agents is very high. With a lower proportion of H-types, the financial markets cease to operate because of adverse selection and lack of funds.

We summarize the above results in the following proposition:

PROPOSITION 1:

a. *The equilibrium is autarky if the level of initial wealth is sufficiently low and efficient if the level of initial wealth is sufficiently high.*

b. *In the intermediate range of initial wealth ($A \in [\hat{A}, \overline{A}]$), the equilibrium is efficient in a wealth constrained economy, and inefficient in a non-wealth constrained economy.*

c. *The threshold level of wealth that prevents the market from collapsing and the threshold level of wealth that yields an efficient equilibrium are higher in a non-wealth constrained economy than in a wealth constrained one.*

d. *Multiple equilibria can only exist between* $\hat{A}(I/p_H R_H)$ *and* $\hat{A}$.

Proposition 1 shows that not only the individual wealth constraint but also the aggregate one matters. Most clearly this can be seen from parts b) and c) of Proposition 1: In a wealth constrained economy, $A \geq \hat{A}$ is a sufficient condition for an efficient equilibrium whereas in a non-wealth constrained economy, it is only a sufficient condition to avoid a collapse of the market to autarky.

Proposition 1 also suggests that changes in agents' wealth may also change the type of equilibrium. A decrease in wealth may shift the economy from an efficient equilibrium to an inefficient one (e.g., from $H^e L^{fs}$ to $H^e L^{efs}$), or even to autarky (e.g., from $H^{ef} L^f$ to autarky). Interestingly, in our model, also an increase in wealth may reduce efficiency: Increasing wealth may move an economy from an efficient $H^{ef} L^f$ equilibrium to an inefficient $H^e L^{ef}$ (from point 1 to point 2 in Figure 2).[12]

Another implication of Proposition 1 is that there is no need for financial intermediaries producing information in the efficient equilibria. However, Boyd and Prescott (1986) show that efficiency-improving financial intermediation could endogenously arise in the range of parameter values where equilibrium $H^e L^{ef}$ prevails. Clearly, in all other inefficient equilibria there is, too, a room for intermediaries mitigating informational problems but the question of whether they can endogenously emerge beyond $H^e L^{ef}$ is left for the future research.

[12] Nonetheless, such a wealth increase can increase social welfare. Up to the diagonal $h=A/I$ the wealth increase improves welfare as it boosts H-type entrepreneurship, but to the extent the increase continues past the diagonal, it reduces efficiency. As a result, whether the net welfare effect of the individual wealth increase is positive or negative depends on the relevant parameter values and the distance of points 1 and 2 from the diagonal. Roughly speaking, if the starting point (point 1) is sufficiently close to diagonal but the ending point (point 2) extends well beyond the diagonal, the increase in initial wealth will also decrease welfare besides efficiency.

Proposition 1 establishes two regions of multiple equilibria. One of these occurs when the proportion of H-type agents is very high. It supports three equilibria, of which one is efficient. In this area $H^{ef}L^{e}$ has a high cost of borrowing (R_B^*) while $H^{ef}L^{e}$ has a low cost of borrowing. The equilibrium with a high cost of borrowing cannot exist to the right of $\hat{A}$ as L-type agents' incentive (and/or) participation constraints would be violated due to their pledgeable income becoming smaller than that of H-types'. The existence of $H^{ef}L^{ef}$ in the same area occurs as L-types can also mix between entrepreneurship and becoming financiers. The $H^{ef}L^{f}$ equilibrium cannot exist to the left of $\hat{A}(I/p_H R_H)$ as the incentive compatibility constraint of L-types would then be violated and they would like to become entrepreneurs.

3.3. Comparison to the Case with Unlimited Supply of Financial Capital

Let us briefly compare our results to those emerging from the standard partial equilibrium models where aggregate wealth is not an issue.[13] In these models there is typically free entry of financiers with unlimited access to financial capital (but without a project of their own), making the supply of funding perfectly elastic.

In their seminal paper, Stiglitz and Weiss (1981) consider mean-preserving spread of project return distributions. In our notation it would mean that $p_H R_H = p_L R_L \equiv \overline{R}$ and, consequently, the L-type entrepreneurs' pledgeable income would exceed the one of the H-type entrepreneurs for all relevant parameter values (i.e., for $A < \hat{A} = \overline{R}$). Hence, it is not surprising that we obtain Stiglitz-Weiss-type results in the region where the aggregate wealth constraint does not bind and $A < \hat{A}$: Financial markets collapse when the proportion of H-type entrepreneurs is low

[13] A more detailed comparison can be found in the working paper version (Takalo and Toivanen 2006).

enough. With a higher proportion of H-types, the quality of the entrepreneurial pool is sufficient to sustain a pooling equilibrium like in Siglitz and Weiss (1981).

In another influential article, de Meza and Webb (1987) assume the first-order stochastic dominance of project return distributions which, in terms of our model, would be equivalent to the assumption of $p_H > p_L$ but $R_H = R_L$. This would make the H-type entrepreneurs' pledgeable income larger than the one of the L-type entrepreneurs for any A (i.e., for $A \geq \hat{A} = 0$). Again, we obtain de Meza-Webb-type overinvestment results in a non-wealth constrained economy when $A \in [\hat{A}, \overline{A}]$: There is too much entrepreneurship as at least some L-types pool with H-types and become entrepreneurs. If the entrepreneurs have a high stake in their projects ($A > \overline{A}$), an efficient separating equilibrium emerges where all L-types invest in storage.[14]

In sum, the standard partial equilibrium models suggest that the financial markets are inefficient because competitive financiers with unlimited investment funds drive the interest rates too low, which encourages unproductive entrepreneurship. Similarly, when the aggregate wealth constraint does not bind in our model, lending and borrowing rates are relatively low, which not only encourages entrepreneurship but also discourages its finance. In contrast, when the aggregate wealth constraint binds, the relative scarcity of funds raises the interest rates, which can be conducive for financial market efficiency in particular in the intermediate individual wealth range ($A \in [\hat{A}, \overline{A}]$). When $A \geq \hat{A}$, the H-type entrepreneurs' pledgeable income exceeds the one of the L-type entrepreneurs. In that case the higher opportunity cost of entrepreneurship discourages foremost L-type entrepreneurs, improving the quality of the entrepreneurial pool. The same logic does not necessarily apply when $A < \hat{A}$, because there the pledgeable income of L-type entrepreneurs is

[14] This is reminiscent of de Meza and Webb (1990) where a separating equilibrium emerges when entrepreneurs are sufficiently risk-averse.

higher than that of H-types'. The higher opportunity cost first affects H-types' choice, causing an adverse effect on the average quality of entrepreneurs.

4. Wealth and Entrepreneurship

The aggregate wealth constraint also affects the relationship between an individual agent's wealth and entrepreneurship. In a non-wealth constrained economy increases in individual wealth may raise the economy out of autarky and cause efficient exit of L-type entrepreneurs in equilibrium $H^{e}L^{efs}$. However, it also stimulates inefficient entry of L-type entrepreneurs in equilibrium $H^{e}L^{ef}$ (see Section 3.1). In a wealth constrained economy individual wealth is positively associated with efficient entry of H-type entrepreneurs in all equilibria except in $H^{ef}L^{ef}$ where H-type entrepreneurs are replaced by L-types as wealth rises. Nonetheless, entrepreneurship in $H^{ef}L^{ef}$ is in aggregate increasing in wealth.

The relationship between an agent's wealth and entrepreneurship can be summarized as follows:

PROPOSITION 2:

a. *An agent's wealth and entrepreneurship are (weakly) negatively correlated if storage is used. In this case, increases in individual wealth lead to efficient exit.*
b. *An agent's wealth and entrepreneurship are (weakly) positively correlated if storage is not used. In this case, increases in individual wealth lead to inefficient entry in a non-wealth constrained economy and efficient entry in a wealth constrained economy when $A \geq \hat{A}$.*

The negative relationship between individual wealth and entrepreneurship arises here for the same reason as in the partial equilibrium models: as the stake in the own project rises, investing in storage rather than own project becomes more attractive for agents with low quality projects. In our model this can happen only if the level of aggregate wealth is so high that some agents prefer to opt out of the financial markets. If all assets of the economy are invested in the entrepreneurial projects, an individual agent's wealth and entrepreneurship are positively correlated. Moreover, increases in individual wealth can lead to entry of H-type entrepreneurs if the economy-level wealth constraint binds.

Note that the attractiveness of the storage technology as an investment option depends on its efficiency. In the limit when there is no storage technology, financial market participate on must be complete by assumption and individual wealth and entrepreneurship must be positively correlated (see Takalo and Toivanen 2006).

5. Policy Implications

Though there are several limitations[15] to our simple model, we boldly offer some policy recommendations concerning the tricky subject of promoting entrepreneurship. Policymakers often view access to finance as one of the key problems facing start-ups (see, e.g., the European Commission 2003, 2008, 2009, the UK Government 2008). Our findings refines the argument advanced by de Meza and Webb (1987 and 1999) that the problems of accessing to finance do not necessarily constitute a reason to subsidize entrepreneurs or their financiers. In our model there is too much lending and entrepreneurship in the intermediate individual wealth range ($A \in [\hat{A}, \overline{A}]$ when the aggregate wealth constraint of an economy is not binding. This applies even if business creation is increasing in the level of individual wealth. However, when we consider the same intermediate wealth range of a wealth constrained economy, it turns out that productive entrepreneurs are held back by insufficient individual wealth.

[15] For instance, future work should consider more than two types of agents, heterogeneity in agents' wealth, financiers' coalition formation, richer contracting space, and more dynamic environment with capital accumulation via consumption and saving decisions.

Moreover, insufficient individual wealth can lead to autarky both in wealth constrained and non-constrained economies. Hence a case for subsidies may arise.

To fix ideas, we could interpret the wealth constrained region of our model as representing capital-constrained emerging economies with investment opportunities, and the non-wealth constrained region as developed countries with cash but with lack of opportunities. In that case our model suggests that the direct public funding of entrepreneurship is more likely to work in emerging economies than in the richer world. The same applies to investment subsidies to financiers, although one should recall that we do not allow for formation of financial intermediaries providing information. While such sophisticated financiers should, as indicated by Boyd and Prescott (1986), improve efficiency at least in the intermediate individual wealth range of non-wealth constrained (developed) economies, they could increase rather than decrease the problems of accessing to finance, as entrepreneurs with bad projects are denied finance.[16]

Subsidies are not the only policy tool to encourage entrepreneurship. For example, the European Commission (2008) advances ten principles for a European small and medium-sized enterprise (SME) policy. Besides the aforementioned "Facilitate SME's access to finance" the principles include goals like "Help SMEs to benefit more from the opportunities offered by the Single Market", "Promote the upgrading of skills in SMEs", and "Encourage and support SMEs to benefit from the growth of markets". Our analysis provides a more optimistic view of the prospects of these initiatives than of the funding interventions in non-wealth constrained (developed) economies. In our model, higher entrepreneurial quality always improves welfare and efficiency, even if it leads to an aggregate wealth constraint: Keeping an agent's wealth constant, an

[16] Moreover, adverse selection can create excessive entrepreneurial entry even in the presence of specialized start-up financiers (see Keuschnigg and Bo Nielsen 2007).

increase in h either yields more successful projects within the initial equilibrium or results in a more efficient equilibrium. Similarly, our model suggests that increases in success probabilities or profits conditional on success are generally conducive for welfare, although they may lead to in efficiencies some circumstances.

6. Conclusions

We study whether, despite asymmetric information and capital constraints, the markets for entrepreneurial finance can endogenously emerge in equilibrium, and the efficiency of the eventual markets. In our model all agents have investment opportunities but encounter capital constraints. They can choose whether they invest their personal wealth in their own project, in others' ventures, or in the storage technology. We identify an economy-level wealth constraint as an important determinant of market efficiency. When the constraint is binding, it reduces the supply of credit and raises its cost. This creates advantageous selection where the agents with productive projects become entrepreneurs and those with unproductive ones become their financiers. In contrast, when credit is plentiful, low interest rates tend to result in overinvestment.

In our model business creation and individual wealth can positively correlated, but this does not necessarily provide a rationale for subsidizing entrepreneurs or their financiers. This result is similar in spirit to de Meza and Webb (1999), but we do not need to invoke moral hazard considerations. We also find that it might be beneficial to subsidize business creation when the aggregate wealth constraint is binding.

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Figure 1 $(H^{e}L^{ef})$

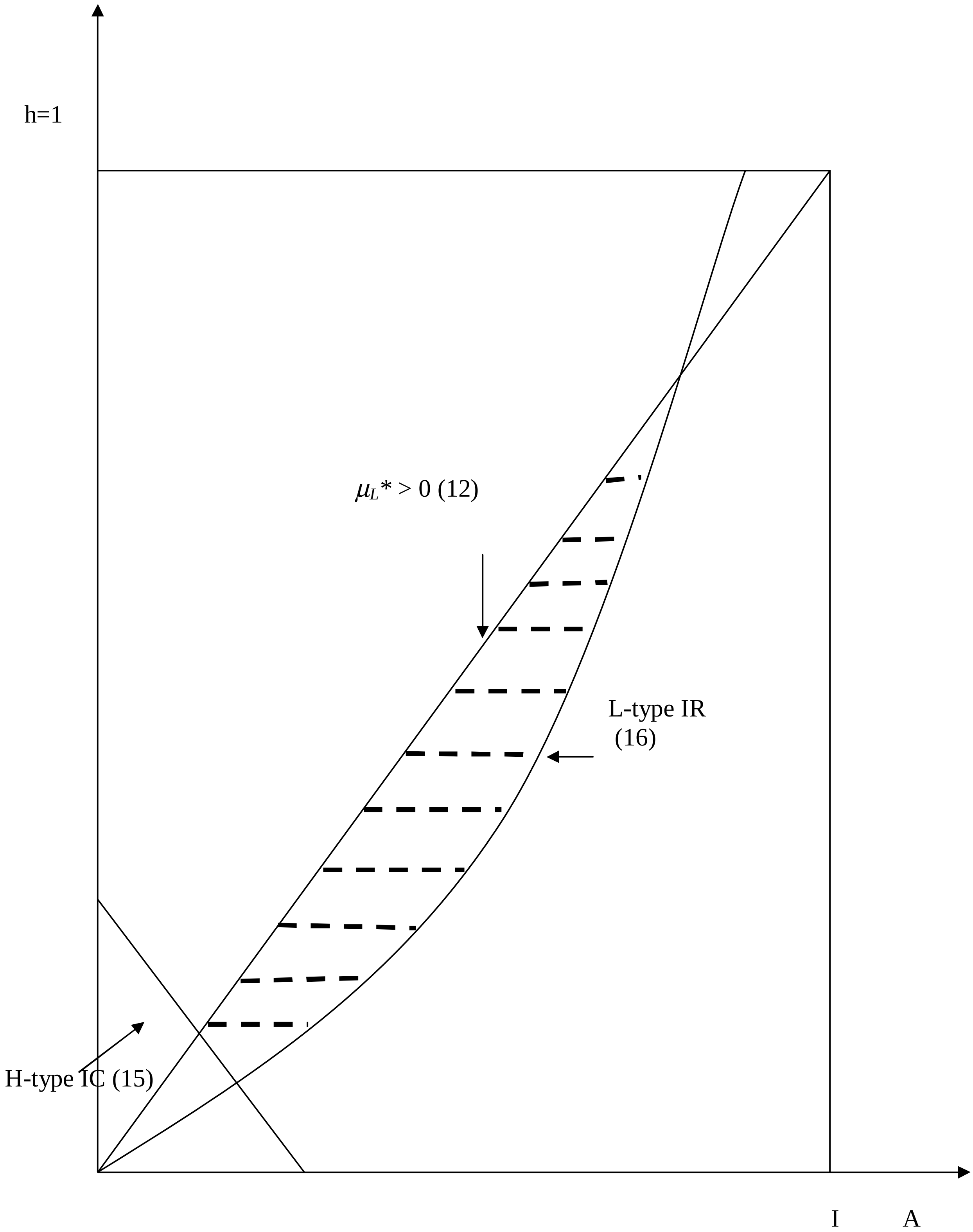
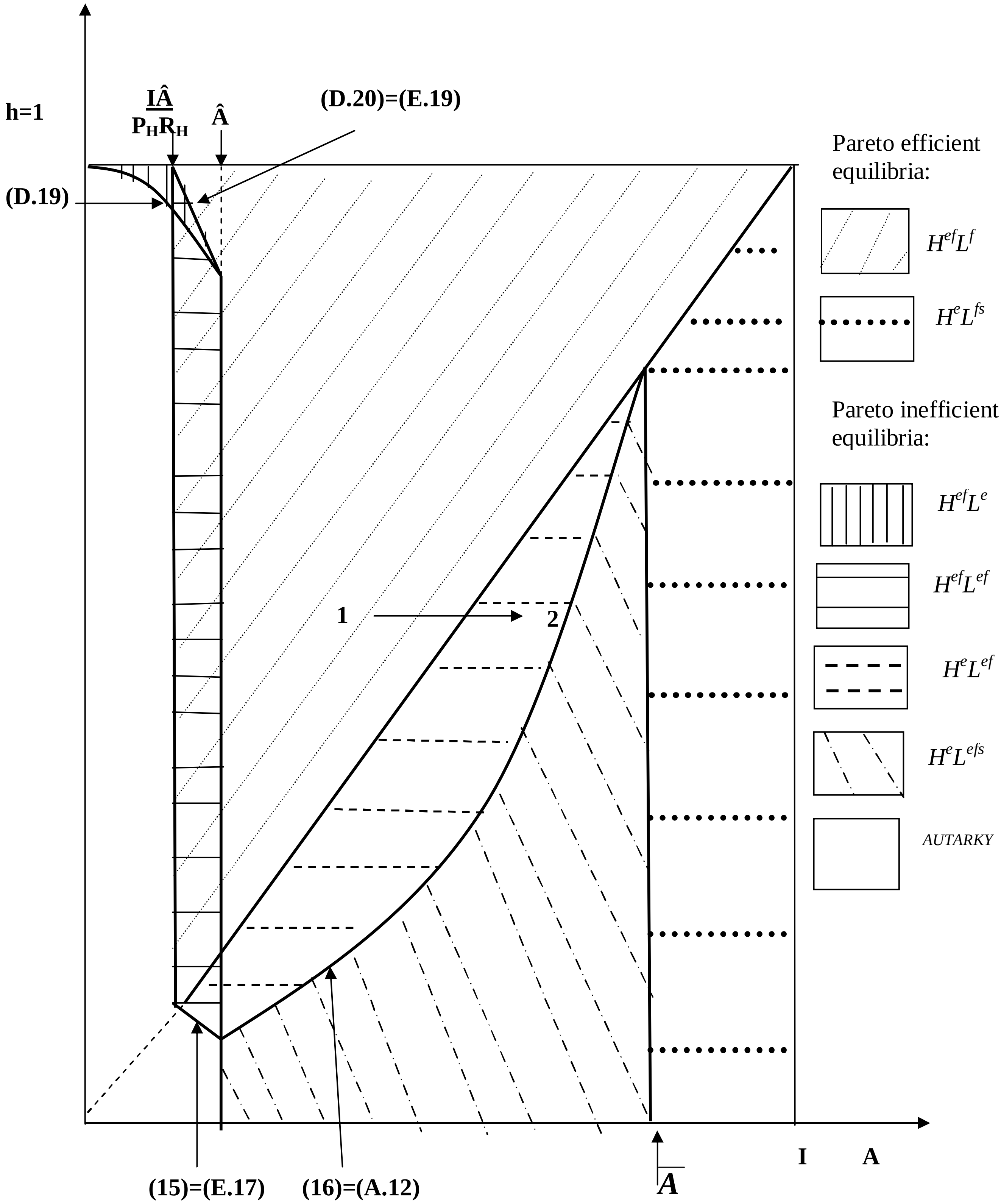


Figure 2



Superscript *e* = entrepreneurship

f = financiers

s = storage technology

Appendices

In these Appendices, we go through all possible equilibria besides autarky. For each equilibrium, we present the constraints, the equilibrium values of endogenous variables, and the equilibrium existence conditions. We shorten the exposition by using the following notation: $\Delta p \equiv p_H - p_L$, $\Delta R \equiv R_L - R_H$, $\gamma \equiv p_H R_H - I$, $\lambda \equiv I - p_L R_L$, and $\Delta W \equiv \gamma + \lambda = p_H R_H - p_L R_L$. Since our approach to solve the model is rather mechanical, we solve for the first equilibrium with providing more details than in subsequent cases.

Appendix A: $H^e L^{ef}$ and $H^e L^{efs}$

$H^e L^{ef}$ is described in Section 3.1, so we here characterize $H^e L^{efs}$ and explain its relation to $H^e L^{ef}$. In $H^e L^{efs}$, all H-types are entrepreneurs and L-types are indifferent among entrepreneurship, financing, and using the storage technology, i.e., $\mu_H=1$, $\mu_L \in (0,1)$, $\chi_H=0$, and $\chi_L \in (0,1)$. That is, the only difference to $H^e L^{ef}$ is that χ_L is strictly positive. Hence the agents' IR constraints (7) must hold as an equality, i.e.,

$$R_F = A. \tag{A.1}$$

The agents' IC constraints are as in (8) and (9), i.e.,

$$p_L (R_L - R_B) = R_F\,, \tag{A.2}$$

and

$$p_H (R_H - R_B) \geq R_F\,. \tag{A.3}$$

The economy level "budget constraint" (10) becomes

$$(1-\mu_L-\chi_L)(1-h)A_{agg} = [h+\mu_L(1-h)](I_{agg}-A_{agg}) \tag{A.4}$$

and, similarly, the equilibrium condition of repayment flows (11) is transformed to

$$[hp_H + \mu_L(1-h)p_L]R_B = R_F(1-\mu_L-\chi_L)(1-h)\,. \tag{A.5}$$

A system consisting of (A.1), (A.2), (A.4) and (A.5) determines the values of the endogenous variables R_B, R_F, χ_L, and μ_L. By (A.1), $R_F^*=A$. Using this in (A.2) and solving it for R_B gives

$$R_B^* = \frac{p_L R_L - A}{p_L}. \tag{A.6}$$

Upon inserting (A.1) and (A.6) into (A.5) and then solving (A.4) and (A.5) for the remaining two endogenous variables, χ_L, and μ_L. yields

$$\mu_L^* = \frac{h}{1-h}\left[\frac{(p_L R_L - A)\Delta p}{p_L \lambda} - 1\right] = \frac{h}{1-h}\left(\frac{R_B^* \Delta p}{\lambda} - 1\right) \tag{A.7}$$

and

$$\chi_L^* = \frac{1}{1-h}\left[1 - \frac{(p_L R_L - A)\Delta p I h}{p_L \lambda A}\right] = \frac{1}{1-h}\left[1 - \frac{R_B^* \Delta p I h}{\lambda A}\right], \tag{A.8}$$

where the last equalities come from (A.6).

The equilibrium exists if χ_L* and μ_L* as given by (A.7) and (A.8) satisfy our initial assumptions $\mu_L \in (0,1)$ and $\chi_L \in (0,1)$, and if the agents' IC and IR constraints are satisfied with R_B* as given by (A.6). The first four existence conditions are

$$\mu_L^* < 1 \iff A > p_L\left(R_L - \frac{\lambda}{h\Delta p}\right), \tag{A.9}$$

$$\mu_L^* > 0 \iff A < p_L\left(R_L - \frac{\lambda}{\Delta p}\right) = \hat{A} + \frac{p_L \gamma}{\Delta p} = \overline{A}, \tag{A.10}$$

$$\chi_L^* < 1 \iff A < \frac{p_L R_L I \Delta p}{I\Delta p + p_L \lambda} = \frac{p_L R_L I \Delta p}{p_H I - p_L^2 R_L}, \tag{A.11}$$

and

$$\chi_L^* > 0 \iff A > \frac{p_L R_L I h \Delta p}{I h \Delta p + p_L \lambda} = \frac{p_L R_L I h \Delta p}{I(p_L + h\Delta p) - p_L^2 R_L}. \tag{A.12}$$

Since L-type IC and IR bind by (A.1) and (A.2), the fifth existence condition comes from H-type IC (A.3). If it is satisfied, H-type IR (A.1) also trivially holds. Inserting (A.1) and (A.6) into (A.3) shows that H-type IC holds if

$$A \geq \hat{A}. \tag{A.13}$$

Equations (A.9)-(A.13) define the range of parameters for which $H^e L^{efs}$ exists. Since the critical values of A in (A.11) and (A.12) are strictly larger than the respective critical values in (A.10) and (A.9), the binding critical values are given by (A.10) and (A.12). They in turn cross each other at the diagonal $h=A/I$. This means that $H^e L^{efs}$ only exists in a non-wealth constrained economy. In terms of the (A, h)-space, $H^e L^{efs}$ exists in the area between the vertical lines (A.13) and (A.10), and below the curve (A.12) (which is identical to equation (16)), as depicted in Figure 2.

When (A.12) is violated, the H-type IC changes from (A.13) to (15). Thus, $H^e L^{ef}$ exists in the range of parameters described in Section 3.1, i.e., in the area shaped by curve (A.12), the downward sloping line (15) and the $h=A/I$-diagonal. The vertical Â-line, (A.12, and (15) cross at the same point where $h=h_1 \equiv \hat{A}\lambda / I\Delta W$.

Appendix B: $H^{ef}L^{f}$ and $H^{efs}L^{fs}$

We first prove that $H^{efs}L^{fs}$ cannot exist. In this equilibrium μ_L=0 and μ_H , χ_H and $\chi_L \in (0,1)$. The equilibrium is constrained by the following five conditions:

$$R_F \geq A. \text{ "L- and H-type IR"} \tag{B.1}$$

$$p_L(R_L - R_B) \leq R_F, \text{ "L-type IC"} \tag{B.2}$$

$$p_H(R_H - R_B) = R_F, \text{ "H-type IC"} \tag{B.3}$$

$$[(1-\mu_H - \chi_H)h + (1-\chi_L)(1-h)]A_{agg} = \mu_H h(I_{agg} - A_{agg}), \tag{B.4}$$

"Equality of supply and demand for funds."

and

$$h\mu_H p_H R_B = R_F[[(1-\mu_H - \chi_H)h + (1-\chi_L)(1-h)]]. \tag{B.5}$$

"Financial market transactions"

In $H^{efs}L^{fs}$ (B.1) holds with equality. Solving (B.4) for μ_H yields

$$\mu_H = \frac{A}{hI}[1-\chi_H h - \chi_L(1-h)] \tag{B.6}$$

Using (B.3) and (B.6) in (B.5) yields R_B as

$$R_B^* = \frac{R_H(I-A)}{I}. \tag{B.7}$$

Inserting (B.7) into (B.3) gives

$$R_F^* = p_H R_H \frac{A}{I}. \tag{B.8}$$

Since R_F^* in (B.8) is strictly larger than A, the initial assumption that (B.1) binds is invalid. This means that $H^{efs}L^{fs}$ cannot exist.

However, $H^{ef}L^{f}$ allows for the inequality in (B.1.). This equilibrium can be characterized by setting $\chi_H = \chi_L$=0 in (B.6). As a result,

$$\mu_H^* = \frac{A}{hI}. \tag{B.9}$$

Equation (B.9) gives two equilibrium existence conditions:

$$\mu_H^* < 1 \Leftrightarrow A < hI, \tag{B.10}$$

and

$$\mu_H^* > 0 \;\Leftrightarrow\; A > 0. \quad \text{(B.11)}$$

By means of (B.7) and (B.8) the L-type IC constraint (B.2) can be written as

$$A \geq \frac{I}{p_H R_H} \hat{A}. \quad \text{(B.12)}$$

Equations (B.10) and (B.12) define the range of parameters for which $H^{ef}L^f$ exists. As shown in Figure 2, the equilibrium exists in a wealth constrained economy for $A \in [\hat{A}I/p_H R_H, I)$.

Appendix C: H^eL^{fs}

In this equilibrium, μ_H =1, μ_L=0, χ_H =0, and $\chi_L \in (0,1)$. The five basic conditions constraining the equilibrium are

$$R_F = A, \text{ "L-type IR"} \quad \text{(C.1)}$$

$$p_L(R_L - R_B) \leq R_F, \text{ "L-type IC"} \quad \text{(C.2)}$$

$$p_H(R_H - R_B) \geq R_F, \text{ "H-type IC and IR"} \quad \text{(C.3)}$$

$$(1-\chi_L)(1-h)A_{agg} = h(I_{agg} - A_{agg}), \quad \text{(C.4)}$$

"Equality of supply and demand for funds"

and

$$hp_H R_B = R_F(1-\chi_L)(1-h). \quad \text{(C.5)}$$

"Financial market transactions"

By substituting the equilibrium value of R_F from (C.1) into (C.5), the other endogenous variables, χ_L, and R_B, can be solved from (C.4) and (C.5). They are given by

$$\chi_L^* = \frac{A - hI}{A(1-h)} \quad \text{(C.6)}$$

and

$$R_B^* = \frac{I - A}{p_H}. \quad \text{(C.7)}$$

From (C.6) we see that χ_L^* <1 by assumption $A<I$. Similarly, inserting (C.1) and (C.7) into (C.3) shows that H-types' IC and IR constraints are equivalent to $p_H R_H > I$ which holds by assumption. Thus, H^eL^{fs} is defined by two existence conditions. First,

$$\chi_L^* \geq 0 \;\Leftrightarrow\; A \geq hI \quad \text{(C.9)}$$

must hold. Second, the L-type IC constraint (C.2) must hold. By employing (C.1) and (C.7), (C.2) can be rewritten as

$$A \geq \hat{A} + \frac{p_L \gamma}{\Delta p} = \overline{A}, \tag{C.10}$$

where the right hand side equals (A.10). Equations (C.9) and (C.10) show that $H^e L^{fs}$ only exists in a non-wealth constrained economy for $A \in [\overline{A}, I)$ (see Figure 2).

Appendix D: $H^{ef}L^e$ and $H^{efs}L^e$

We first prove that $H^{efs}L^e$ cannot exist. In $H^{efs}L^e$, $\mu_H \in (0,1)$, μ_L=1, $\chi_H \in (0,1)$ and χ_L=0. The five basic constraints in $H^{efs}L^e$ are

$$R_F = A, \text{ "L- and H-type IR"} \tag{D.1}$$

$$p_L(R_L - R_B) \geq R_F, \text{ "L-type IC"} \tag{D.2}$$

$$p_H(R_H - R_B) = R_F, \text{ "H-type IC"} \tag{D.3}$$

$$(1 - \mu_H - \chi_H)hA_{agg} = [1 - h + \mu_H h](I_{aff} - A_{agg}), \tag{D.4}$$

"Equality of supply and demand for funds",

and

$$[h\mu_H p_H + (1-h)p_L]R_B = R_F(1 - \mu_H - \chi_H)h. \tag{D.5}$$

"Financial market transactions"

A system consisting of (D.1) and (D.3)-(D.5) determines the values of the endogenous variables R_F, R_B, χ_H, and μ_H. Since R_F*=A by (D.1), (D.3) gives R_B as

$$R_B^* = \frac{p_H R_H - A}{p_H}. \tag{D.6}$$

Upon substituting (D.1) and (D.6) into (D.5) and somewhat involved algebra, (D.4) and (D.5) can be written as

$$\mu_H^* = \frac{1-h}{h}\left[1 - \frac{\Delta p(p_H R_H - A)}{p_H \gamma}\right] = \frac{1-h}{h}\left(1 - \frac{R_B^* \Delta p}{\gamma}\right) \tag{D.8}$$

and

$$\chi_H^* = \frac{1}{h}\left[1 - \frac{\Delta p(p_H R_H - A)I(1-h)}{p_H \gamma A}\right] = \frac{1}{h}\left[1 - \frac{R_B^* \Delta p I(1-h)}{\gamma A}\right] \tag{D.9}$$

Equations (D.8) and (D.9) provide four equilibrium existence conditions:

$$\mu_H^* \le 1 \;\Leftrightarrow\; A \le \frac{p_H\left[h\gamma + (1-h)\lambda\right]}{\Delta p(1-h)}, \tag{D.10}$$

$$\mu_H^* \ge 0 \;\Leftrightarrow\; A \ge \frac{p_H (I - p_L R_H)}{\Delta p}, \tag{D.11}$$

$$\chi_H^* \le 1 \;\Leftrightarrow\; A \le \frac{p_H R_H I\Delta p}{I\Delta p + p_H\gamma} = \frac{p_H R_H I\Delta p}{p_H^2 R_H - p_L I}, \tag{D.12}$$

and

$$\chi_H^* \ge 0 \;\Leftrightarrow\; A \ge \frac{p_H R_H I(1-h)\Delta p}{I(1-h)\Delta p + p_H\gamma} = \frac{p_H R_H I(1-h)\Delta p}{p_H^2 R_H - I(p_L + h\Delta p)}. \tag{D.13}$$

Since H-types' IC and IR bind, and L-types' IR is satisfied through their IC, the L-type IC (D.2) is the fifth equilibrium existence condition. By using (D.1) and (D.6), we see that (D.2) is satisfied if

$$A \le \hat{A}. \tag{D.14}$$

Since the vertical line (D.14) is smaller in value than the vertical line (D.11), the equilibrium cannot exist for positive χ_{H}.

In contrast, $H^{ef}L^e$ does exist. To see this, note first that in $H^{ef}L^e$, (D.1) should be rewritten as an inequality $R_F \ge A$. Then, let χ_H=0 in (D.4) to get

$$\mu_H^* = \frac{A - (1-h)I}{hI}. \tag{D.15}$$

Substituting (D.15) and (D.3) for (D.5) and letting χ_H=0 yields

$$R_B^* = \frac{p_H R_H (I - A)}{I(p_L + h\Delta p)}. \tag{D.16}$$

Inserting (D.16) back into (D.3) gives

$$R_F^* = \frac{p_H R_H\left[p_H A - \Delta p I(1-h)\right]}{I(p_L + h\Delta p)}. \tag{D.17}$$

From (D.15) we see that $\mu_H^* < 1$ holds by our assumption that $A<I$. For $\mu_H^* \ge 0$ we need that

$$A \ge (1-h)I. \tag{D.18}$$

The H-type IR is now $R_F \ge A$, which - using (D.17) - can be expressed as

$$A \geq \frac{p_H R_H I\Delta p(1-h)}{I\Delta p(1-h)+p_H\gamma} = \frac{p_H R_H I\Delta p(1-h)}{p_H^2 R_H - I[p_L + h\Delta p]}. \tag{D.19}$$

Similarly, by means of (D.16) and (D.17) the L-type IC (D.2) is given by

$$A \leq I - \frac{I\Delta W(p_L + h\Delta p)}{\Delta p p_H R_H} = \frac{I}{p_H R_H}\left[\hat{A} + \Delta W(1-h)\right]. \tag{D.20}$$

Conditions (D.18)-(D.20) characterize the existence of $H^{ef}L^{e}$. Equation (D.18) is a downward sloping h=1-A/I-diagonal that starts from the (A=0, h=1) corner and ends in the (A=I, h=0) corner. H-types' IR constraint (D.19) is a monotonically downward sloping curve that starts from the (A=0, h=1) corner and cuts the h=1-A/I-diagonal once after the vertical $\hat{A}$-line. The L-type IC constraint (D.20) is a downward sloping line that begins from the vertical $\hat{A}I/p_HR_H$ -line (when h=1), cutting the h=1-A/I-diagonal after the vertical $\hat{A}$-line. H-types' IR and L-types' IC constraints and the vertical $\hat{A}$-line cross at the same point at $h=h_2\equiv 1-\hat{A}\gamma/\mathrm{I}\Delta W$, which is above the h=1-A/I-diagonal. This renders (D.18) redundant. In sum, $H^{ef}L^{e}$ exists above the H-type IR curve (D.19) and below the L-type IC line (D.20). This area is in the upper-left corner of the (A, h)-space where $A\in[0, \hat{A}]$ and $h\in[h_2, 1]$ (see Figure 2).

Appendix E: $H^{ef}L^{ef}$ and $H^{efs}L^{efs}$

We first prove that $H^{efs}L^{efs}$ cannot exist for a non-trivial set of parameters. In this equilibrium, all μ_H , μ_L, χ_H , and $\chi_L \in (0,1)$. The agents' IR and IC constraints bind, i.e., it must hold that

$$R_F = A, \text{ "L- and H-type IR"} \tag{E.1}$$

$$p_L(R_L - R_B) = R_F, \text{ "L-type IC"} \qquad \text{(E.2),}$$

and

$$p_H(R_H - R_B) = R_F. \text{ "H-type IC"} \tag{E.3}$$

Solving (E.2)-(E.3) for R_B gives

$$R_B^* = \frac{\Delta W}{\Delta p}. \tag{E.6}$$

Thus, there is a unique value of

$$A = p_H(R_H - \frac{\Delta W}{\Delta p}) = p_L(R_L - \frac{\Delta W}{\Delta p}) = \hat{A}. \tag{E.7}$$

for which this equilibrium can exist. This means that only $H^{ef}L^{ef}$ (where both μ_H and $\mu_L \in (0,1)$ but $\chi_H=\chi_L=0$) may exist for a non-trivial range of parameters.

$H^{ef}L^{ef}$ is constrained by the following five basic conditions:

$$R_F \geq A, \text{"L- and H-type IR"} \tag{E.8}$$

$$p_L(R_L - R_B) = R_F, \text{"L-type IC"} \tag{E.9}$$

$$p_H(R_H - R_B) = R_F, \text{"H-type IC"} \tag{E.10}$$

$$[(1-\mu_H)h + (1-\mu_L)(1-h)]A_{agg} = [\mu_L(1-h) + \mu_H h](I_{agg} - A_{agg}), \tag{E.11}$$

"Equality of supply and demand for funds"

and

$$[(1-\mu_H)h + (1-\mu_L)(1-h)]R_F = [p_L\mu_L(1-h) + p_H\mu_H h]R_B. \tag{E.12}$$

"Financial market transactions"

Equation system (E.9)-(E.12) determines the values of the endogenous variables, R_F, R_B, μ_L, and μ_H. Solving (E.9) and (E.10) for R_B and R_F gives

$$R_B^* = \frac{\Delta W}{\Delta p} \tag{E.13}$$

and

$$R_F^* = p_H(R_H - \frac{\Delta W}{\Delta p}) = p_L(R_L - \frac{\Delta W}{\Delta p}) = \hat{A}. \tag{E.14}$$

Substituting (E.13) and (E.14) into (E.12) and solving (E.11) and (E.12) for μ_L and μ_H yields

$$\mu_H^* = \frac{1}{h\Delta W}\left(\hat{A} - \frac{Ap_L R_L}{I}\right) \tag{E.15}$$

and

$$\mu_L^* = \frac{1}{(1-h)\Delta W}\left(\frac{Ap_H R_H}{I} - \hat{A}\right). \tag{E.16}$$

Equations (E.15) and (E.16) yield four equilibrium existence conditions:

$$\mu_H^* < 1 \iff A > \frac{I}{p_L R_L}\left(\hat{A} - h\Delta W\right), \tag{E.17}$$

$$\mu_H^* > 0 \iff A < \frac{I}{p_L R_L}\hat{A}, \tag{E.18}$$

$$\mu_L^* < 1 \iff A < \frac{I}{p_H R_H}\left[\hat{A} + (1-h)\Delta W\right], \tag{E.19}$$

and

$$\mu_L^* > 0 \iff A > \frac{I}{p_H R_H}\hat{A}. \tag{E.20}$$

Equations (E.8) and (E.14) imply that that agents' IR constraints are satisfied if

$$A \leq \hat{A}. \tag{E.21}$$

This is the fifth equilibrium existence condition. However, we see that if condition (E.21) holds, (E.18) also holds. The equilibrium is thus defined by equations (E.17), and (E.19)-(E.21). Since (E.19) is identical to (D.20) we know that it cuts the vertical $\hat{A}$-line at $h=h_2$ where $h_2 \equiv 1-\hat{A}\gamma/I\Delta W$ as defined in Appendix D. This means that when h is large enough, i.e, $h \in [h_2, 1]$, the downward sloping line (E.19) and the vertical line (E.20) are the binding constraints. For $h \in [h_3, h_2]$ where $h_3 \equiv \hat{A}/p_H R_H$, the binding constraints are the vertical lines (E.20) and (E.21). For $h \in [h_1, h_3]$, where $h_1 \equiv \hat{A}\lambda/I\Delta W$ as defined in Appendix A, the binding constraints are (E.17) (which is identical to (15)) and (E.21). For $h<h_1$, the equilibrium does not exists, since (E.17) is violated.

In Figure 2 we illustrate how in terms of the (A, h)-space, $H^{ef}L^{ef}$ exists in a parallelogram between the vertical lines (E.20) and (E.21) and the downward sloping lines (E.17) and (E.19). This parallelogram exists for $A \in (\hat{A}I/p_H R_H, \hat{A})$ and $h \in [h_1, 1]$.