Optimal fragmentation of intellectual property rights*

Klaus Kultti
Department of Economics, University of Helsinki

and

Tuomas Takalo
Research Department, Bank of Finland

Abstract

We develop an equilibrium model of product innovation to study the implications of independent invention for the design of intellectual property rights. In the model agents choose whether to be innovators seeking new ideas or imitators absorbing spillovers, and multiple innovators can find the same idea. It turns out that the optimal intellectual property right is typically strong but non-exclusive, involving fragmentation of the right among different innovators. The optimal number of property right holders is inversely related to the cost of innovation and obsolescence rate. Exclusive patent protection can be approximately optimal only if innovation is costly and the obsolescence rate is high.

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Klaus Kultti, Department of Economics, P.O. BOX 54, FIN-00014 University of Helsinki. klaus.kultti@helsinki.fi. Tuomas Takalo, Bank of Finland, Research Department, P.O. Box 160, FIN-00101 Helsinki. tuomas.takalo@bof.fi. We thank Vincenzo Denicolò and two referees for extensive comments. We also thank Eli Berman, Jan Eeckhout, Glenn Harma, Samuel Kortum, Emmi Martikainen, Mikko Mustonen, Jennifer Reinganum, Susanne Scotchmer, Samuli Simojoki, Rune Stenbacka, Juuso Toikka, Mihkel Tombak, Juuso Välimäki and participants in numerous seminars and conferences for criticisms of earlier drafts. A part of this research was conducted when Takalo was visiting the IDEI, Toulouse, and the Economics Departments at the Massachusetts Institute of Technology and Boston University. We gratefully acknowledge financial support from the Academy of Finland and the Yrjö Jahnsson Foundation.

1. Introduction

The possibility of multiple independent discoveries of the same idea is ubiquitous in research in different industries (see, e.g. Granstrand, 1999, pp. 25 and 52, Armond, 2003, and Lemley and Chien, 2003). This especially seems to characterise network industries such as software, the internet, telecommunications and payment systems, where standardisation limits the possible paths for future technologies and so firms concentrate their R&D activities on the same fields. Similar views are expressed by Rahnasto (2003) and Varian, Farrell and Shapiro (2004). Rahnasto (2003), in particular, argues that simultaneous innovation has resulted in fragmentation of intellectual property rights (IPRs) in these industries. As a result, products and services in network industries are based on combinations of IPRs that are held by multiple owners. Because of the hold-up power conferred to a single patent holder, it has been suggested that fragmentation retards the use of innovations. The discussion has, however, mainly been concerned with vertical fragmentation in the chain of innovations.

We develop an equilibrium model of product innovation to study the *horizontal* fragmentation of IPRs, which means that more than one inventor of the same technology obtains the right to use it. Because such fragmentation renders the market for new products more competitive, it should stimulate the use of innovations rather than retard it. We find that optimal IPRs are strong and perpetual but typically non-exclusive. The cheaper the innovation, the larger the number of property right holders that should be granted for an innovation. Exclusive patent protection is approximately optimal only if innovation is very costly.

Related to optimal fragmentation of IPRs is the question of whether independent invention should be a defence to infringement. A key distinction between patents and other forms of IP protection is that only one patent can be awarded even in the case of several independent developments of the same idea, whereas other forms recognise the rights of independent inventors. In the seminal analysis of independent invention with IPRs Wright (1983) advocates “marginal patents” that are awarded only in the case of unduplicated innovation, but regards such awards as practically impossible to implement. Closest to our work is La Manna, MacLeod and de Meza (1989), which presents a patent race model with free entry. They advocate a permissive patent system allowing independent infringements of patents over a given period of time and show that it is welfare-superior to the traditional strict patent system under a wide range of circumstances. A reason is that in the permissive system wasteful duplication of innovation costs is less of an issue, since more innovators leads to more production. This effect is present in our model, but there is also an additional benefit, since more innovators produce a greater number of new products.

Another difference compared with La Manna et al. (1989) is that in our model the set of agents is well-specified, and the agents become either innovators who seek ideas or imitators who enjoy spillovers. Our analysis can be seen to provide a robustness check for their partial equilibrium results, because in our model innovators and imitators must fare equally well in equilibrium. This corresponds to the free entry condition of La Manna et al. (1989). Moreover, we distinguish innovations from ideas, consider multiple innovations, clarify how innovators happen to discover the same innovation independently, and determine the expected number of innovators to discover the same innovation. We characterise optimal IP protection, including the optimal number of property right holders, and perform comparative statics analysis.

More recent studies of the effects of independent invention on the design of IP policy include Bessen and Maskin (2002) and Maurer and Scotchmer (2002).[1] In Maurer and Scotchmer (2002) a patent holder can be threatened by rivals' independent duplication, and such duplication is a defence to infringement. The patent holder, facing an unlimited number of potential licensees but no competing licensors, has an incentive to grant a license to deter duplication. In our model independence of invention is different, as there is no possibility of consciously duplicating already existing innovations.[2] It is thus somewhat awkward to investigate licensing, because there may be several patent holders and several (but a limited number of) independent inventors, i.e., potential licensees. To determine the price of the license in this environment would require careful modelling of competing sellers and buyers.

Bessen and Maskin (2002) provide a tractable model of cumulative innovation where in each period two firms compete to make an improvement on an existing innovation. They compare patents to a system without IPRs. Since the firms must obtain access to the original innovation to be able to make an improvement, patents create hold-up power over subsequent innovations. Thus, a system without IPRs may generate more innovation than the patent system. Our work can be considered complementary to Bessen and Maskin (2002). Although we do not allow for cumulative innovation, we do consider the optimal number of property right holders over one generation of innovation. Even in the absence of cumulative innovation, it is in general optimal to have multiple property right holders to foster competition in the market for new products.

We present the model in section 2. In section 3 we determine the optimal form of

[1] These and our study also exemplify 'independent invention'; we became aware of each others' studies after having completed them independently.

2 In an extension, Maurer and Scotchmer (2002) use the same notion of independent invention as La Manna et al. (1989) and we do. The extension can be obtained as a special case of our model.

an IPR. The concluding remarks are in section 4.

2. The model

2.1 Assumptions

We consider an infinite horizon, discrete time economy with A agents who live forever. The agents can choose between two activities called innovation and imitation. Innovating is an uncertain activity in which agents seek ideas. The way agents discover ideas is based on the urn-ball model familiar from the literature on random matching (see, e.g., Wolinsky, 1988, and Lu and McAfee, 1996). There are I ideas, and the agents randomly and independently contact them. We assume that A and I are large, so that the (binomially distributed) number of agents that discover an idea can be approximated by a Poisson distribution with rate $\theta = \frac{Ax}{I}$, where x is the share of agents who innovate. To each idea corresponds an inverse demand function of the form $p = \sqrt{2} - q$.

All agents who discover the same idea produce perfect substitutes and behave as Cournot-competitors.[3] The demand for various innovations is symmetrically distributed among the agents in the economy, and since the number of agents is large no innovator takes into account her own demand in deciding how much to produce. The innovators must bear a periodic cost $c > 0$ to turn an idea into an innovation. The marginal cost of production is zero. The assumptions about costs and demand-form imply that the social value of a discovered idea – an innovation - is unity.

We introduce three variables for IP policy. One is the strength of protection, α, which determines the probability that the IPR is valid and cannot be infringed, i.e., with

[3] As our complementary study (Kultti, Takalo, and Toikka, 2003) also suggests, the main findings of this paper extend to product market competition with prices, rather than quantities, as the strategic variable.

probability 1-α an innovation becomes public, so that anyone can use it for free, and with probability α it remains proprietary. Another is the duration of the IPR, T, which is the number of periods for which the IPR for an innovation is enforced. After T periods the innovation becomes public with certainty. The novelty here lies in the third policy variable, which is the number of property right holders, n; the larger it is, the more fragmented the IPR becomes. We assume that whenever an innovation is made, exactly n agents get a property right to it. We elaborate this assumption in the next section.

An innovation can also become obsolete, i.e., the demand for it will cease, each period with probability 1-λ, and with probability λ it continues to be viable. This means that there are three types of innovations: proprietary, public, and obsolete. Obsolete innovations are economically useless. Innovators derive utility from the profits on their proprietary innovations and from consumer surplus generated by both proprietary and public innovations. Imitators receive only consumer surplus.

The events within a period occur in the following order: 1) In the beginning the innovators incur a cost c; 2) The innovators come up with an idea, which then constitutes an innovation; 3) An innovation becomes obsolete with probability 1-λ and public with probability 1-α; 4) Production takes place according to the static Cournot model, where output depends on the number of property right holders n. [4]

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[4] Our findings would be qualitatively similar if the events with probabilities α and λ were realised after production.

2.2 Discussion and interpretation

The interpretation of $\theta = \frac{Ax}{I}$ is somewhat tricky. It determines the probability of making an innovation, but it is endogenous, as the agents split up into innovative and imitative sectors. The number of ideas, I, is however a fixed parameter, which indicates the economy's innovative or productive potential.

The distinction between ideas and innovations in our model resembles that of O'Donoghue, Scotchmer and Thisse (1998), who assume that it is costly to turn an idea into an innovation. They assume that ideas arrive at a firm at an exogenous rate. They and Kortum (1997) endogenise the rate at which ideas become obsolete, while we treat it as exogenous. In our model its main function is to guarantee the existence of a steady state.

As mentioned, we interpret α as the strength of IP protection. In each period, an innovation remains in the possession of IPR-holders with probability α and becomes public with probability $1-\alpha$. Thus we adhere to the probabilistic view of IPRs (Lemley and Shapiro, 2005).[5] Once in the public domain, the innovation is available for free to all agents, and competition ensures that production is at the level where price equals marginal cost. Then profit is zero and consumer surplus unity.

When both α and T are policy variables but protection of an innovation is given exclusively to one innovator regardless of the number of actual innovators (n=1), the form of protection is much like a patent. There are many ways in which a patent can become useless before its formal expiration - 20 years after application, in most countries. For example, patents are vulnerable to court challenges to their legal validity and they can be

[5] As a referee noted, in our case α differs from the breadth of protection, because breadth refers to whether a similar but non-identical technology can be sold in the market. We cannot easily address this notion of breadth in our model since a competing product is either in the same market as the proprietary product or in a different market, with no substitutability in demand. Thus identifying α with strength makes more sense.

forfeited via failure to renew, properly reserve rights, or observe the various other legal formalities. Patents can also be revoked in connection with re-examination or opposition, and the protected technology may move to the public domain when competitors manage to invent around the patent. The rights of a patent holder can also be violated if this is beneficial (e.g., repair and research exemptions) or necessary (compulsory licensing) to the society as a whole. Thus real-world policies affecting α abound; these include disclosure requirements, renewal fees, preliminary injunction practices, litigation costs, burden of proof, and procedural limits to court challenges.

When the number of IPR-holders, n, is greater than unity, IP protection is not exclusive. We determine the socially optimal value of n. For reasons of tractability, we assume that when an idea is discovered, protection is granted to exactly n innovators, regardless of the number of independent innovators. If there are k innovators who discover the same idea, and $k > n$, all k innovators have an equal chance of becoming one of n who are protected. If $k < n$, all k innovators receive protection for certain and $n - k$ other agents in the innovative sector are randomly awarded protection.

If protection is always given to a predetermined number of innovators, issues associated with independent-invention defence seem to no longer be highly relevant. But we emphasise that the rule is adopted for technical tractability. This is practically equivalent to a rule that gives the maximum number of independent innovators, say m, who can get protection. This rule yields the innovators certain expected profits, which can be approximated by our rule that grants protection to exactly n innovators. Compared to the rule by which at most m receive protection, our rule provides more profit if we set n=1, as then production is at monopoly level. But if n=m, our rule provides less profit. Between 0 and m, there is some value of n that gives innovators approximately the same expected profit as does the rule by which at most m receive protection. Because agents' decisions depend solely on expected profits, the two rules result in approximately the same outcome.[6] In section 3.2 we advance a practical way to implement the rule where exactly n receive protection.

Finally, even without IPRs innovations can be protected to an extent by keeping them secret. In a discussion paper version (Kultti and Takalo, 2003), we consider an economy without IPRs, where all who discover the same idea can produce the innovation and α measures the strength of secrecy in the absence of legal protection. With probability α, an innovation can be kept secret; otherwise there is a leak and the innovation becomes public knowledge. In so far there is no leakage, innovations can remain secret forever. This benchmark economy includes standard distortions arising from incomplete appropriability of the social value of innovation (due to the absence of perfect price discrimination and spillovers), market power in the post-innovation market, and business stealing. As a rule, the equilibrium size of the innovative sector differs from the social optimum in the benchmark economy, and this is the rationale for considering IPRs in our model.

2.3 Equilibrium

We focus on a steady state. To that end, we assume that ideas turned into innovations are replaced by new ideas, so that their number remains the same. An agent's only decision is whether to become innovator or imitator. Because agents' demands are symmetrically distributed over the innovations, each agent receives the same amount of consumer surplus. Innovators also profit from proprietary innovations, but in equilibrium investing c in innovative activity must yield a return equal to c. This determines the proportion of agents

[6] The approximation is not exact, as n is an integer. But the mistake is small and hardly changes our results quantitatively - qualitatively not at all.

who are innovators, x, and the equilibrium is fully determined once the numbers of innovators and imitators are known. As a result, the equilibrium condition is given by

$$\pi_n\left[\sum_{k=0}^{n-1} e^{-\theta}\frac{\theta^k}{k!}+\sum_{k=n}^{\infty} e^{-\theta}\frac{\theta^k}{k!}\frac{n}{k+1}+\frac{I}{Ax}\sum_{k=1}^{n-1} e^{-\theta}\frac{\theta^k(n-k)}{k!}\right]\sum_{t=1}^{T}(\alpha\lambda)^t - c = 0\,. \tag{1}$$

The left-hand side of (1) gives the expected profit of an imitator entering the innovative sector.[7] There $\pi_n = \dfrac{2}{(n+1)^2}$ denotes the per-period profit from a proprietary innovation with n property right holders. In the first term in brackets, $e^{-\theta}\dfrac{\theta^k}{k!}$ is the probability that an innovator discovers an idea along with exactly k other innovators. If $k \le n-1$, the innovator receives for certain the IPR and associated profit. If there are more than n-1 other innovators, the innovator gets the IPR with probability $\dfrac{n}{k+1}$, which is in the second term in brackets. The last term in brackets gives the expected number of IPRs that the innovator receives for free over all other innovations with less than n innovators. The term outside the brackets is the sum of the probabilities that the innovation remains proprietary, i.e., becomes neither public nor obsolete for $t \in \{1,2,\dots,T\}$ periods.

Applying some algebra, (1) simplifies to

$$\left(\frac{1-e^{-\theta}}{\theta}\right)\frac{\alpha\lambda}{1-\alpha\lambda}\left(1-(\alpha\lambda)^T\right)\frac{2n}{(n+1)^2} = c\,. \tag{2}$$

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[7] In deriving (1) we assume that the agent's discount factor is unity. Besides greatly simplifying the analysis, the assumption it is harmless since the discount factor plays a minor role. We are only interested in situations where investment today for a future stream of payoffs is reasonable. Because the innovations can become public and obsolete, the discount factor should be close to unity.

The equilibrium size of the innovative sector has the properties one expects:

LEMMA. $dx/d\alpha > 0$, $dx/dT > 0$, $dx/dn < 0$, $dx/dc < 0$, and $dx/d\lambda > 0$.

Proof: See appendix.

The first three properties mean that the stronger, longer, and less fragmented the IPR, the larger the innovative sector. The last two show that the size of the innovative sector is inversely related to the cost of innovation and obsolescence rate.

2.4. Welfare

Our social welfare measure comprises the aggregate consumer surplus and aggregate net profit. Since we focus on the steady state and since the agents live forever, it is sufficient to consider welfare in a period instead of in an infinite stream. Hence, to determine the aggregate welfare, we need only to account for the steady state distributions of proprietary and public innovations and related profits and consumer surpluses.

Let ω_t denote the stock of proprietary innovations of age $t \in \{1,2,...,T\}$ after it is known which innovations become public or obsolete. In each period, $\omega_0 \equiv \left(1-e^{-\theta}\right)I$ innovations are discovered, so that $\omega_1 = \alpha\lambda\omega_0$. Because in each period an innovation remains proprietary with probability $\alpha\lambda$, the stock of proprietary innovations of age t is $\omega_t = (\alpha\lambda)^t \omega_0$. Thus the steady state stock of proprietary innovations is given by

$$\sum_{t=1}^{T} \omega_t = \left(1-e^{-\theta}\right)I \frac{\alpha\lambda}{1-\alpha\lambda}\left(1-(\alpha\lambda)^T\right). \tag{3}$$

Consider next the stock of public innovations, denoted by *P*. To find its steady state value, we determine flows into and from the stock. In each period, a proprietary innovation becomes public with probability $1-\alpha$ and, with probability λ, does not become obsolete. Thus the inflow is $(1-\alpha)\lambda\sum_{t=0}^{T-1}\omega_t + \lambda\omega_T$, where the last term is the stock of innovations whose IPRs expire. The outflow consists of innovations that become obsolete, i.e., $(1-\lambda)P$. From the equality of the flows and some manipulation we obtain

$$P = \frac{(1-e^{-\theta})I\lambda}{1-\alpha\lambda}\left[\frac{1-\alpha}{1-\lambda} + \alpha(\alpha\lambda)^T\right]. \tag{4}$$

Each proprietary innovation generates a per-period profit of $\pi_n = \frac{2}{(n+1)^2}$ and consumer surplus of $\gamma_n = \left(\frac{n}{n+1}\right)^2$, whereas each public innovation yields zero profit but full (unity) consumer surplus. Combining appropriate per-period profit and consumer surplus with steady state stocks, the welfare function can be expressed as $W = P + (\gamma_n + n\pi_n)\sum_{t=1}^{T}\omega_t - Axc$ or using (3) and (4), as

$$W = \frac{(1-e^{-\theta})I\lambda}{1-\alpha\lambda}\left\{\left[\frac{1-\alpha}{1-\lambda} + \alpha(\alpha\lambda)^T\right] + \alpha\left[1-(\alpha\lambda)^T\right](\gamma_n + n\pi_n)\right\} - Axc. \tag{5}$$

In (5), the first term comes from the full consumer surplus of the public innovations, the second term is the sum of consumer surplus and profit of proprietary innovations, and the last term is the total cost of innovation.

3. Optimal IP protection

3.1 Analysis and results

As shown in section 2.3, in equilibrium innovators' return equals to zero. Thus we can insert (2) into (5) and rewrite equilibrium welfare as

$$W = \frac{\left(1-e^{-\theta}\right)I\lambda}{1-\alpha\lambda}\left\{\left[\frac{1-\alpha}{1-\lambda}+\alpha\left(\alpha\lambda\right)^{T}\right]+\alpha\left[1-\left(\alpha\lambda\right)^{T}\right]\left(\frac{n}{n+1}\right)^{2}\right\}. \tag{6}$$

Equation (6) shows how equilibrium welfare equals aggregate consumer surplus. To find the optimal IP protection, we differentiate (6) with respect to α, T, and n, taking into account that x in $\theta = \frac{Ax}{I}$ is a function of policy variables via (2). The first-order conditions for optimal strength, duration, and fragmentation of the IPR can, after some algebra, be written as

$$\Phi(\alpha,T,n) \equiv \frac{\theta e^{-\theta}(1-\alpha\lambda)}{\alpha(1-\lambda)\left(1-(\alpha\lambda)^{T}\right)} - \frac{2n+1}{(n+1)^{2}}\left(1-e^{-\theta}\right) = 0. \tag{7a}$$

$$-\Phi(\alpha,T,n)(\alpha\lambda)^{T+1}\ln\alpha\lambda = 0 \tag{7b}$$

and

$$\frac{2n^{2}}{n+1}\left(1-(\alpha\lambda)^{T}\right)\left(1-e^{-\theta}\right) - \theta e^{-\theta}\left[n^{2}-(\alpha\lambda)^{T}+\frac{(n^{2}-1)(1-\alpha)}{\alpha(1-\lambda)}\right] = 0. \tag{7c}$$

We proceed under the assumption that (7a)-(7c) characterise the maximum. Later, in the proof of Proposition 3, we show that the optimal IPR actually involves a corner solution for α, but this has no impact on the conclusions based on (7a)-(7c).

PROPOSITION 1. There is a continuum of socially optimal combinations of strength and duration of IPRs for any positive *n*.

This observation is immediate from (7a) and (7b). It is also a special case for Denicolò (1996), who shows that if static welfare and incentive to innovate are linear in patent breadth, total welfare is independent of the combination of patent breadth and duration. In our model the strength has non-linear effects on both static welfare and size of innovative sector, as can be seen from (5) and (2). The effects of strength and duration are identical in terms of expected values so that they are perfect policy substitutes.[8] Because of this we can, without loss of generality, characterise an optimal patent system by determining the optimal value of α when $n=1$ and $T\to\infty$.[9]

PROPOSITION 2. When patent duration is infinite, the optimal patent strength is given by

$$\alpha^* = \min\left\{1, \frac{4\theta e^{-\theta}}{4\lambda\theta e^{-\theta} + 3(1-\lambda)(1-e^{-\theta})}\right\}.$$

[8] The effect of demand is captured by the term containing *n* in (2) and (5), which is independent of the way α and *T* appear in the equations. The size of the innovative sector also affects the incentive to innovate and welfare, and it is monotone in α, *T*, and *n*. Thus this and subsequent results should hold for all demand functions with such a monotonicity property.

[9] Although $T\to\infty$, patented innovations do not remain profitable forever, because they become public or obsolete. Following O'Donoghue et al. (1998), we could interpret $1/(1-\alpha\lambda)$ as the effective patent life. Evidence indicates (e.g., Schankerman and Pakes, 1986, and Lanjouw, 1998) that effective life is typically much shorter than a statutory patent term, which emphasises the relevance of patent strength.

Proof: See appendix.

Note that the optimal strength at an interior solution is given in implicit form, since the size of the innovative sector x in θ is an endogenous variable and hence a function of α^*. The optimal strength balances the standard tradeoffs associated with patent protection. On the one hand, strengthening patent protection stimulates innovative activity and leads to more innovations, which increases consumer surplus. On the other hand, more active innovation increases the number of innovations discovered by several innovators independently. This entails wasteful duplication of innovation cost, since only one of the innovators is granted an exclusive right to the innovation. As usual, stronger patents also curb consumer surplus per innovation. In contrast to the standard case, it is possible here that the optimal patent protection is perfect, i.e., that $\alpha^* = 1$ (and $T \to \infty$). This happens if the cost of innovation or the obsolescence rate is exceptionally high, so that the innovative sector becomes very small. In such circumstances the possibility of multiple discovery is negligible, and each agent in the innovative sector is likely to produce a unique innovation.

The optimal strength varies with the cost of innovation, c, and obsolescence rate, $1-\lambda$. The cost of innovation has only an indirect effect on α^* via the size of the innovative sector. Because α^* is decreasing in x, which in turn is decreasing in c (by the Lemma), *the strongest patents should be granted where innovation is costly*. The effect of the obsolescence rate is less clear. The indirect equilibrium effect is positive, since a high obsolescence rate requires strong patent protection to restore the incentive to innovate. But a high obsolescence rate also has a negative direct effect on the optimal strength, since it makes innovations less valuable to society as a whole, so there is less need to stimulate innovation.

Our main result is that an exclusive IPR such as that of Proposition 2 is typically inefficient.

PROPOSITION 3. The optimal IPR has maximum strength and maximum duration, and in general is fragmented among multiple property right holders. Formally, the optimal IPR is given by

$$\alpha^* = 1,\ T^* \to \infty, \text{ and } n^* = \frac{2\left(1 - e^{-\theta}\right)}{\theta e^{-\theta}} - 1 \geq 1.$$

Proof: See appendix.

Proposition 3 says that the strength and duration of protection should be maximal, whereas the right to produce the innovation should typically be given to several innovators. The number of property right holders is simply a more efficient means of balancing the standard tradeoffs articulated above than is the strength (or duration) of an IPR. The reason is twofold. First, in the case of multiple discovery of an idea, any exclusive system inherently involves inefficient duplication. Fragmenting the IPR among several innovators increases production, which makes duplication less wasteful. Second, the benefits of a weak IPR spill equally into the innovative and imitative sectors. This encourages free riding more than fragmentation, which gives relatively larger rewards to innovators. A given increase in welfare can be achieved with less damage to the incentive to innovate by fragmenting the IPR than by diluting its strength.[10]

We can determine the expected number of innovators to discover the same innovation and accordingly the optimal number of IPR holders. Here it is again important to note that the optimal number given by Proposition 3 is in implicit form. Because x in θ

[10] The second reason goes back to the standard ratio test (see, e.g., Denicolò, 1996): with linear demand, the ratio of static deadweight loss to static profit is smaller under oligopoly than under monopoly.

is determined in equilibrium, n^* also indirectly depends on the cost of innovation, c, and the obsolescence rate, $1-\lambda$. Combining Proposition 3 with the Lemma then yields

COROLLARY. The lower the cost of innovation or obsolescence rate, the higher the optimal number of property right holders for an innovation, i.e., $dn^*/dc<0$ and $dn^*/d\lambda>0$.

The Lemma suggests that the innovation-incentive effects of cost of innovation, obsolescence rate, and number of property right holders are qualitatively similar. Therefore, if the cost of innovation or obsolescence rate decreases, the market for new products can be made more competitive without affecting the incentive to innovate. It can be shown that n^* is less than two when either c or λ is sufficiently high. Consequently, the patent system is approximately optimal when the cost of innovation or obsolescence rate is sufficiently high.

3.2. Discussion and interpretation

If we take the optimal protection rule literally, it corresponds to a mix of independent-invention defence and mandatory licensing. Admittedly, such a mix is rarely found in practice. But it could be applied in that the defence could be limited to independent invention that is already complete at the time the first patent application becomes public as in La Manna et al. (1989). If there are fewer than n independent inventors, say m, before the first application becomes public, the patent authorities could auction off n-m additional patents. It is likely that the auction winners would be firms working on similar R&D projects. This would also encourage the firms to scrutinise patent information more carefully.

Regardless of the interpretation, solving for the optimal number of innovators to receive protection is valuable, because it tells us how fragmented the markets for innovation should be. As mentioned in the introduction, fragmentation is usually discussed in the context of vertical innovation chains, where it can hinder the use of innovations because of the hold-up power created. In our model, however, one can think of an IPR as being *horizontally* fragmented among innovators of the same idea when multiple permits to use the idea are granted. Such horizontal fragmentation should enhance the use of innovations because the market for new products becomes more competitive.

Besides patent policy, our model could also be used to investigate the effects of other prevailing IPRs. For example, a policy where T is infinite and n equals the number of agents who discover the idea reminds one of trade secret laws. They can be thought of enforcing perpetual IP protection over an innovation, as long as it is not disclosed or nobody invents the innovation independently. In that case, α is a measure of the strength of trade secret protection, i.e., the extent to which innovators are protected against information leaks due to, e.g., breach of security and unlawful employee communication with competitors.

Innovation in our model could also be viewed as a creative activity under copyright protection. A copyright protects against direct copying but not against independent invention. Indeed, a work is eligible for copyright protection only if it is sufficiently independent. Moreover, a copyright does not protect an idea, and its duration is typically restricted to 70 years from the innovator's death. Under the copyright interpretation, a copyright is granted to all independent inventors of an idea for T periods, and α indicates the strength of the copyright as measured by, e.g., how difficult it is to limit or even exhaust a copyright by invoking fair use and first sale doctrines, or by introducing copyrighted products in the public domain, e.g., via the internet; or how much of the copyright owner's rights are exhausted when the work is made available to the public.[11]

4. Conclusion

Independent invention is frequent in practice, perhaps increasingly so. In such an environment, IPRs have complex welfare effects. To examine them, we develop an equilibrium model of product innovation where multiple innovators may discover the same idea. In our model there are two sectors, innovation and imitation, whose sizes are determined in equilibrium.

A key implication of the model is that duplication of innovation cost increases the numbers of new products and potential producers per product. Since the IPR regime determines the actual number of producers per product, it also determines the wastefulness of duplication. Independent of IPR regime, cost duplication is not wasteful if innovators come up with different innovations. But in the case of multiple discovery, fragmenting the IPR among several producers involves less wasteful duplication than any exclusive system. Fragmentation is also a more efficient way to increase production than is reducing the strength of an IPR, because it dilutes the incentive to innovate less. For example, a weak patent system benefits competitors irrespective of whether they are innovators or imitators while fragmentation benefits the innovators more. Thus the optimal IPR is typically non-exclusive. The cheaper the innovation and the lower the obsolescence rate, the higher should be the optimal number of property right holders for an innovation.

[11] On the one hand, the copyright interpretation is appealing, as the occupational choice incorporated in our model is often the main economic decision encountered by potential creators of copyrighted works. On the other hand, the interpretation is somewhat far fetched, as we should think of Cournot-competitors as producing distinct expressions of the same idea and let expressions, rather than ideas, become public.

Bold interpretation of our findings argues against the expansion of patentability of software and business methods. Since software and business method innovations are relatively cheap, independent invention should be a defence to infringement. Our model predicts that the expansion of software and business method patentability not only restricts post-innovation competition but can also weaken the incentive to innovate. According to Bessen and Hunt (2004), this is what the data from software industry indicate.

However, our results do not necessarily suggest abolition of the patent system. The patent system could be modified to recognise the rights of independent inventors or the patent authorities could auction off some additional patents for each innovation. Here our findings support some reforms of preliminary injunction practice and first-to-invent rule of the US patent system, as proposed by Armond (2003) and Lemley and Chien (2003). Because priority disputes often deal with independent inventors, the disputes could be resolved by granting patents to all genuine innovators. We also find that the patent system is approximately optimal if the cost of innovation or obsolescence rate is sufficiently high. The conclusion is well in line with the related literature. In La Manna et al. (1989) the patent system can be superior to the permissive system if the production economics of scale are large. Waterson and Ireland (1998) find that a patent system can dominate copyrighting only if production innovation is expensive and, for Maurer and Scotchmer (2002), independent-invention defence is not welfare enhancing if the cost of innovation is sufficiently higher than the cost of duplication.

There are issues that our model fails to address. Here the size of the innovative sector and number of innovations are determined in equilibrium. This effect is missing in partial equilibrium models of innovation. But in a richer model the demand functions would be derived from utility maximisation subject to given resources. Another important issue beyond the scope of this study is cumulative innovation, as we concentrate on the stationary equilibrium where innovations do not build on previous innovations. Therefore, we miss a dynamic effect, which should be incorporated into any complete model of fragmentation of IPRs. Allowing for sequential innovation and vertical fragmentation would emphasise the problems stemming from the hold-up power gained by a single IPR holder. Such hold-up power of patent holders in the cumulative innovation context is used as an argument against the patent system. That we find arguments against exclusive patent rights without recourse to cumulative innovation is noteworthy.

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Appendix

Proof of Lemma. Totally differentiating the equilibrium condition (2) and using it to simplify the expressions yields $\frac{dx}{d\alpha}=\frac{x\left(1-e^{-\theta}\right)\left[1-(\alpha\lambda)^{T}-(\alpha\lambda)^{T}T(1-\alpha\lambda)\right]}{(1-\alpha\lambda)\alpha\left[1-(\alpha\lambda)^{T}\right]\left(1-e^{-\theta}-\theta e^{-\theta}\right)}>0$,

$\frac{dx}{dT}=-\frac{x\left(1-e^{-\theta}\right)(\alpha\lambda)^{T}\log\alpha\lambda}{\left[1-(\alpha\lambda)^{T}\right]\left(1-e^{-\theta}-\theta e^{-\theta}\right)}>0$, $\frac{dx}{d\lambda}=\frac{x\left(1-e^{-\theta}\right)\left[1-(\alpha\lambda)^{T}(1+T-\alpha\lambda T)\right]}{\lambda(1-\alpha\lambda)\left[1-(\alpha\lambda)^{T}\right]\left(1-e^{-\theta}-\theta e^{-\theta}\right)}>0$,

$\frac{dx}{dc}=-\frac{(n+1)^{2}x\theta(1-\alpha\lambda)}{2n\alpha\lambda\left[1-(\alpha\lambda)^{T}\right]\left(1-e^{-\theta}-\theta e^{-\theta}\right)}<0$, and $\frac{dx}{dn}=-\frac{x(n-1)\left(1-e^{-\theta}\right)}{n(n+1)\left(1-e^{-\theta}-\theta e^{-\theta}\right)}<0$. *QED*

Proof of Proposition 2. By (7a), Φ(α,∞,1)=0 implies that $\frac{\theta e^{-\theta}(1-\alpha\lambda)}{\alpha(1-\lambda)}-\frac{3}{4}\left(1-e^{-\theta}\right)=0$ or, upon rearranging, that $\alpha^{*}=\frac{4\theta e^{-\theta}}{4\lambda\theta e^{-\theta}+3(1-\lambda)\left(1-e^{-\theta}\right)}$. The RHS can be larger than unity for small values of θ. Then the optimal strength should be at a maximum, i.e., α*=1. *QED*

Proof of Proposition 3. We first prove that when (7c) holds, the LHSs of (7a) and (7b) are positive. Equation (7c) be rewritten as

$$\frac{\left(1-(\alpha\lambda)^{T}\right)\left(1-e^{-\theta}\right)\alpha(1-\lambda)}{(n+1)\theta e^{-\theta}}=\frac{\left[n^{2}(1-\alpha\lambda)-(\alpha\lambda)^{T}\alpha(1-\lambda)+\alpha-1\right]}{2n^{2}}. \qquad \text{(A1)}$$

From the definition of Φ(α,T,n) in (7a) we see that Φ(α,T,n)>0 if

$$\left(n+1\right)\left(1-\alpha\lambda\right)-(2n+1)\frac{\left(1-(\alpha\lambda)^{T}\right)\left(1-e^{-\theta}\right)\alpha\left(1-\lambda\right)}{(n+1)\theta e^{-\theta}}>0\,. \qquad \text{(A2)}$$

Inserting the LHS of (A1) into (A2) yields that $\Phi(\alpha,T,n)>0$ if $\left(n+1\right)\left(1-\alpha\lambda\right)-\frac{(2n+1)}{2n^{2}}\left[n^{2}\left(1-\alpha\lambda\right)-(\alpha\lambda)^{T}\alpha\left(1-\lambda\right)+\alpha-1\right]>0$ or if $\frac{\left(1-\alpha\lambda\right)}{2}+\frac{\left(2n+1\right)}{2n^{2}}\left[(\alpha\lambda)^{T}\alpha\left(1-\lambda\right)+1-\alpha\right]>0$, which holds. Consequently, the LHS of (7a) is always positive yielding a corner solution $\alpha^*=1$. The LHS of (7b) is also positive and approaches zero in the limit when $T^*\to\infty$.

Letting $\alpha^*=1$ and $T^*\to\infty$ in (7c) and rearranging yields

$$n^{*}=\frac{2\left(1-e^{-\theta(n^{*})}\right)}{\theta\left(n^{*}\right)e^{-\theta(n^{*})}}-1\,, \qquad \text{(A3)}$$

where we explicitly show that n^* is in implicit form. Since the RHS of (A3) is decreasing in n (by the Lemma) and approaching unity when $n\to\infty$, there is at most one solution to (A3). Since welfare is at a minimum when $n\to\infty$, the solution to (A3) characterizes the maximum. If there is no solution to (A3), the optimal policy must be at a corner solution where $n=1$ and the pair (α, T) is optimally chosen. Since there is a continuum of solutions for (7a) and (7b) (by Proposition 1), we can focus on the optimal patent system of Proposition 2, where $T\to\infty$ and α is optimally chosen.

To complete the proof we note from (A.3) that the condition $n^*\leq 2$ implies that $2\left(1-e^{-\theta}\right)-3\theta e^{-\theta}\leq 0$. The LHS has a unique strictly positive root. Since θ is increasing in x, and x is decreasing in c (by the Lemma), there is a unique $\bar{c}$ such that if $c\leq\bar{c}$, n^* is at least two. That is, if $c\leq\bar{c}$ the fragmented IPR system is welfare superior to the optimal patent system. If $c > \bar{c}$, the optimal value of *n* is at most two and, the optimal IPR is nearly equivalent to a patent system.[12] The above logic applies also to λ when *c* is fixed. *QED*

[12] We have not shown which of the systems yields higher welfare when $n^* < 2$. However, the case with $n^* = 1$, is equivalent to $2 - 2e^{\theta} - 2\theta e^{\theta} = 0$, which never holds. If we let *c* increase so that θ becomes small and $2 - 2e^{\theta} - 2\theta e^{\theta} = 0$ almost holds and, if we insert the equation into the formula of α^* given by Proposition 2, we obtain $\alpha^* = 1$, since the second argument of the min-function is greater than unity. This means that α^* is nearly always unity. The only case that remains to be studied is when the value of $n^* \in (1,2)$. If *n* is treated as an integer, it must be either unity or two in the optimum, which makes it hard to show that α^* is still unity but we conjecture that this is the case.